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(13)

-: HAND WRITTEN NOTES:-

OF

(113)

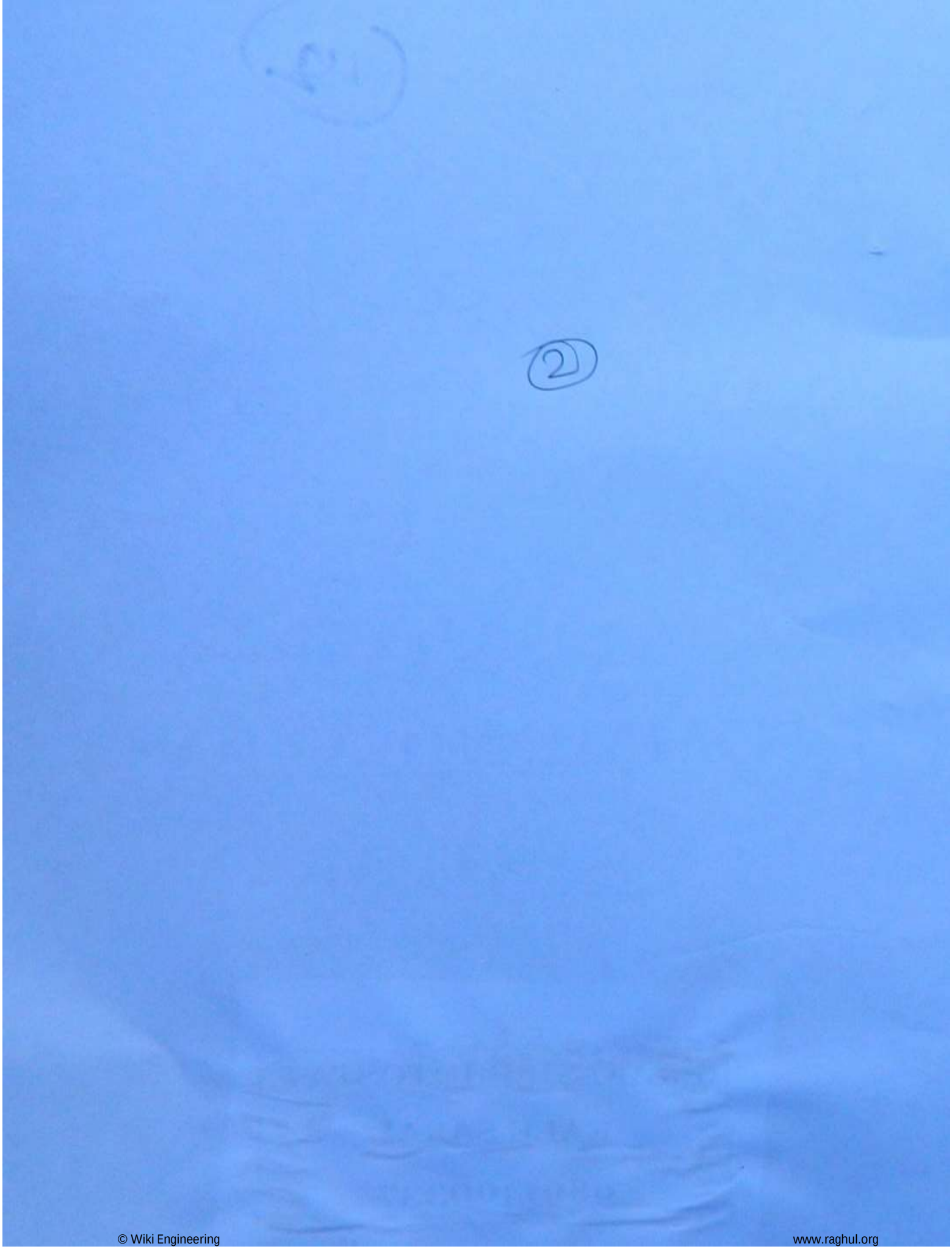
ELECTRICAL ENGINEERING

(1)

-: SUBJECT:-

COMMUNICATION SYSTEM

13



COMMUNICATION ENGINEERING (3)

* Electronic communication is concerned with the transmission of any signal through antenna at relatively high frequencies.

* The audio signal cannot be transmitted over a longer distance since the attenuation of such signals is very fast.

* To transmit the audio signal we translate it to higher frequency components. The translation of these frequencies is then called modulation.

* Once the signal is transmitted through antenna at high frequencies the same signal has to be converted back in the audio range at recieving point. This process is then called the demodulation process. The demodulation is always followed once the modulation of the signal takes place.

Advantages of Modulation

* Long distance ~~comm~~ transmission is possible
* The range of transmission can be increased as per requirement to increase the signal power being transmitted thereby increasing the signal to noise ratio of the system
* Practical length of the antenna is required
* Frequency division multiplexing (FDM) is possible and

Therefore large number of signals can be transmitted simultaneously having different carrier frequency either in the AM range or in the FM range.

(4)

Audio range (AF) : f : 20 Hz - 20 kHz

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{20\times10^3} = 15\times10^3 = 15 \text{ km}$$

$\ell = \frac{\lambda}{2}, \frac{\lambda}{4}$

↳ length of antenna

Electronic communication ⇒ modulation

⇒ translation of freq comp

↓

→ AM broadcast range f : 53F~~H~~z - 1605 kHz

→ FM broadcast range f : 88 MHz - 108 MHz

Modulation

1. Information signal f_m : 20 Hz - 20 kHz

modulating frequency

modulating signal

2. Carrier

... contains no information

... high frequency sinusoidal signal

↳ AM : f = 535 kHz - 1605 kHz

↳ FM : f = 88 MHz - 108 MHz.

Analog Modulation

1. Modulating signal (5)

$f(t)$ — $v_m(t) = V_m \cos \omega_m t$; $\omega_m = 2\pi f_m$

f_m : 20 Hz – 20 kHz

non-sinusoidal : .... single tone modulation

.... multi tone modulation

2. Carrier

$v_c(t) = V_c \cos \omega_c t$

$\omega_c \gg \omega_m$

$= 2\pi f_c$

- AM → 53F KHz – 1605 KHz
- FM → 88 MHz – 108 MHz

AM (Amplitude modulation)

For amplitude modulated signal, the amplitude of the carrier is varied in accordance with instantaneous value of the amplitude of modulating signal keeping the frequency & the phase of the carrier constant

Mathematical expression of AM signal

$v_m(t) = V_m \cos \omega_m t$

$v_c(t) = V_c \cos \omega_c t$

$v_{AM}(t) = (V_c + \underbrace{K_a}_{\text{constant or sensitivity}} v_m(t)) \cos \omega_c t$

$K_a = 1$ unless specified

$$v_{AM}(t) = V_c [1 + K_a V_m \cos \omega_m t] \cos \omega_c t$$

V_c

(6)

m_a ... modulation index

$$[0 \le m_a \le 1]$$

m_a : 0.45 to 0.65

⇒ 45% to 65%

$v_{AM}(t) = V_c (1 + m_a \cos(\omega_m t)) \cos \omega_c t$ ... AM signal

... modulated signal

$$v_{AM}(t) = \underbrace{V_c \cos \omega_c t}_{\text{free carrier (contains no information)}} + \frac{1}{2} m_a V_c \left[\underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}} \right]$$

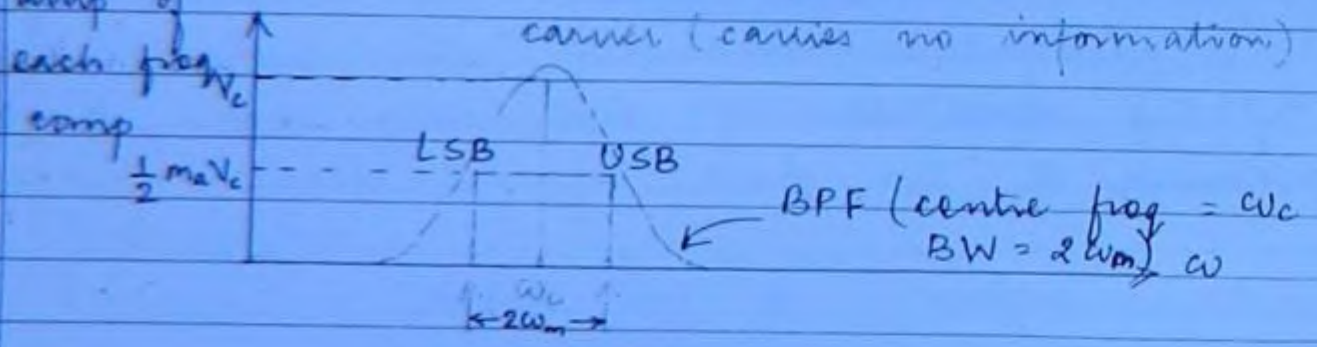
upper side band

lower side band

... AM - DSB/FC

... standard AM signal

Spectrum of AM - DSB/FC signal



The AM - DSB/FC signal represents a standard AM signal & requires maximum amount of power for its generation & maximum amount of bandwidth for its transmission.

* Such system can be utilised for the broadcast AM transmission where there are many recievers corresponding to one transmitter.

* The upper & lower side band contain same information since both side bands are placed symmetrically about the carrier frequency ω_c.

* To generate the standard AM signal the last stage of the system is always a BPF tuned at the centre frequency ω_c having a minimum Bandwidth of $2\omega_m$.

Representation of various AM signal -

AM - DSB / FC

$$v(t) = \underbrace{V_c \cos \omega_c t}_{\text{Carrier}} + \frac{1}{2} m_a V_c \left[\underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}} \right]$$

AM - DSB / SC

$$v(t) = \frac{1}{2} m_a V_c \left[\underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}} \right]$$

AM - SSB / FC

$$v(t) = \underbrace{V_c \cos \omega_c t}_{\text{Carrier}} + \frac{1}{2} m_a V_c \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}}$$

AM - SSB / SC

$$v(t) = \frac{1}{2} m_a V_c \cos(\omega_c + \omega_m)t$$

Power required for AM - DSB/FC

freq	amplitude	power
ω_c	V_c	$\propto V_c^2 = k V_c^2 \equiv P_c$
$\omega_c + \omega_m$	$\frac{1}{2} m_a V_c$	$\propto \left(\frac{1}{2} m_a V_c\right)^2 = \frac{1}{4} m_a^2 V_c^2$ $= \frac{1}{4} m_a^2 P_c$
$\omega_c - \omega_m$	— do —	— do —

$$P_{total} = P_{carrier} + P_{USB} + P_{LSB}$$

$$P_t = P_c + \frac{1}{4} m_a^2 P_c + \frac{1}{4} m_a^2 P_c$$

$$\boxed{P_t = P_c \left(1 + \frac{m_a^2}{2}\right)}$$

↑ unmodulated carrier power

$$I_t^2 R = I_c^2 R \left(1 + \frac{m_a^2}{2}\right)$$

$$\boxed{I_t = I_c \sqrt{1 + \frac{m_a^2}{2}}}$$

$$m_a = \sqrt{m_{a_1}^2 + m_{a_2}^2 + \dots}$$

for more than one modulating signal.

Ex. Find

i) total power required

ii) power saving

iii) % power saving w.r.t the standard AM signal for various types of AM signals.

Assume modulation index of unity.

$m_a = 1$

Case 1 AM - DSB/FC

$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) = \frac{3}{2} P_c$

$P_s = 0$

$\% P_s = 0$

$BW = 2\omega_m$

Case 2 AM - DSB/SC

$P_t = \cancel{P_c}^{0} + P_c \frac{m_a^2}{2} = \frac{1}{2} P_c$

$P_s = P_c$

$\% P_s = \frac{P_c}{\frac{3}{2} P_c} \times 100 \simeq 67\%$

$BW = 2\omega_m$

Case 3 AM - SSB/FC

$P_t = P_c + P_c \frac{m_a^2}{4} + \cancel{P_c \frac{m_a^2}{4}}^{0} = \frac{5}{4} P_c$

$P_s = P_c \frac{m_a^2}{4} = \frac{P_c}{4}$

$\% P_s = \frac{P_c/4}{\frac{3}{2} P_c} \times 100 \simeq 16\%$

$BW = \omega_m$

Case 4 AM - SSB / SC

$$P_t = \cancel{P_c} + P_c\frac{ma}{4} + P_c\frac{ma^2}{4} = \frac{P_c}{4}$$

$$P_s = \frac{P_c}{4} + P_c\frac{ma^2}{4} = \frac{5}{4}P_c$$

$$\%\,P_s = \frac{5/4\,P_c}{3/2\,P_c} \times 100 \cong 83\%$$

$$BW = \omega_m$$

The AM - DSB / FC system requires maximum power for its generation & maximum bandwidth for its transmission but the circuitry required is the simplest. Hence such system is used for broadcast purpose where there are many recievers corresponding to each transmitter.

The AM - SSB / SC system requires minimum power for its generation & minimum Bandwidth for its transmission but the circuitry required is most complex. Therefore such system can be used for point to point communication or military communication where there are one or at the most two recievers corresponding to each transmitter.

Transmission Efficiency -

$$\eta = \frac{\text{useful power}}{\text{total power}} \times 100\%$$

case 1 AM - DSB/FC

$$\eta = \frac{P_{SB}}{P_t} \times 100\% = \frac{P_c \, m_a^2/2}{P_c (1 + m_a^2/2)} \times 100\%$$

$$\eta = \frac{m_a^2}{2 + m_a^2} \times 100$$

for $m_a = 1$

$\eta_{max} \simeq 33\%$

case 2 AM - DSB/SC

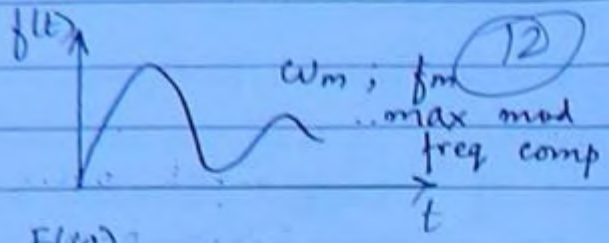
$$\eta = \frac{P_{SB}}{P_t} \times 100\% = \frac{P_c \, m_a^2/2}{P_c (1 + m_a^2/2)} \times 100\%$$

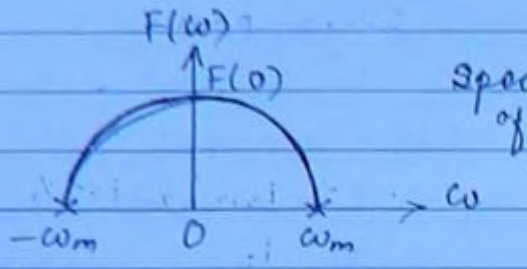
$\eta = 100\%$

* For the standard AM the transmission efficiency depends upon the modulation index. For such system maximum efficiency can be obtained for a max^m modulation index of unity & is only 33%.

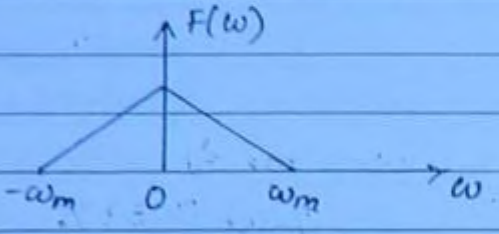
* For suppressed carrier system the efficiency is always 100% irrespective of the value of modulation index.

Multi tone modulation - modulating signal = $f(t)$ ... non sinusoidal





Spectrum of mod signal



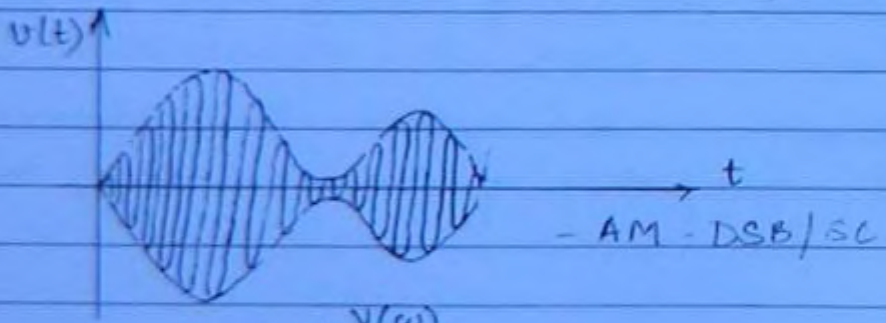
Carrier ... $\cos \omega_c t$

$\omega_c \gg \omega_m$

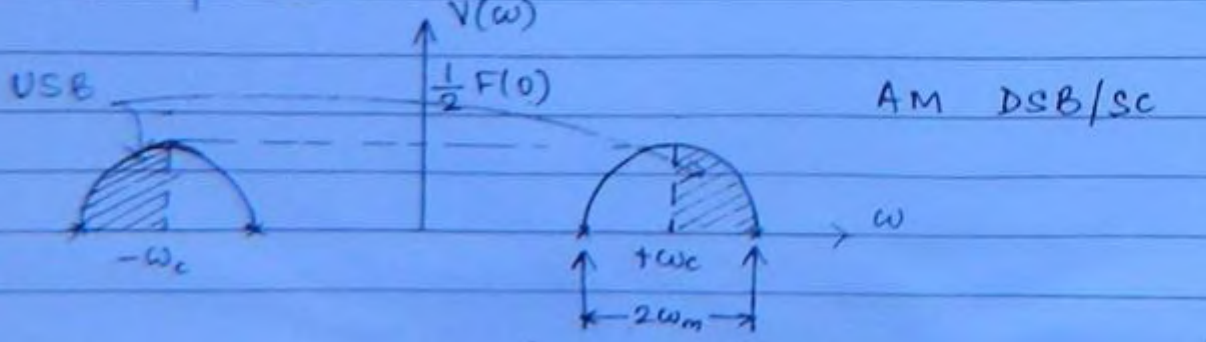
Case I AM DSB/SC

$v(t) = f(t) \cos \omega_c t$ ... AM - DSB/SC

$V(\omega) = \frac{1}{2}[F(\omega + \omega_c) + F(\omega - \omega_c)]$



- AM - DSB/SC



AM DSB/SC

case 2. AM-DSB/FC

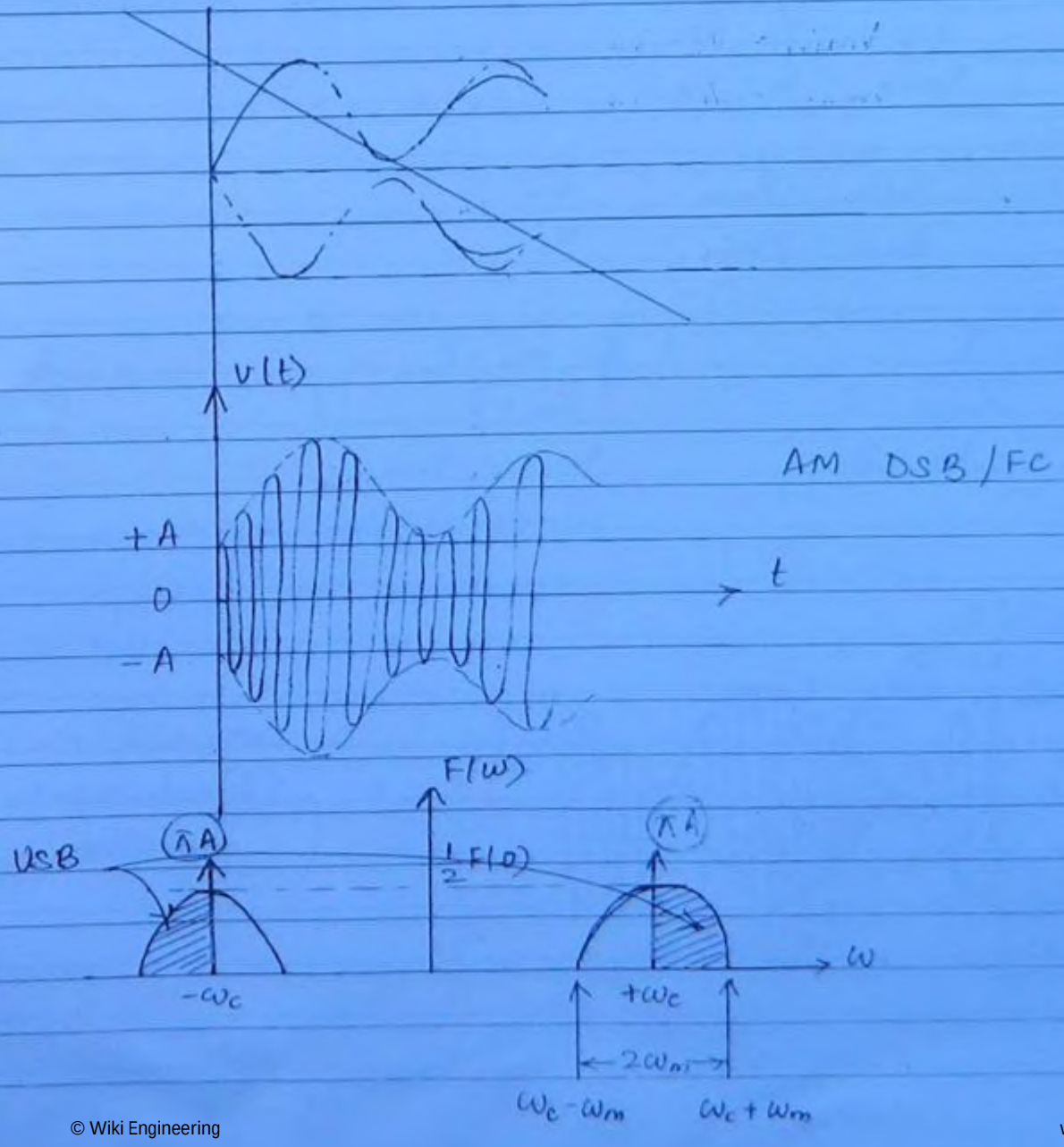
$$v(t) = \underbrace{f(t)\cos\omega_c t}_{\text{AM-DSB/SC}} + \underbrace{A\cos\omega_c t}_{\text{free carrier}}$$

AM-DSB/FC

$$v(t) = (A + f(t))\cos\omega_c t \qquad \text{AM-DSB/FC}$$

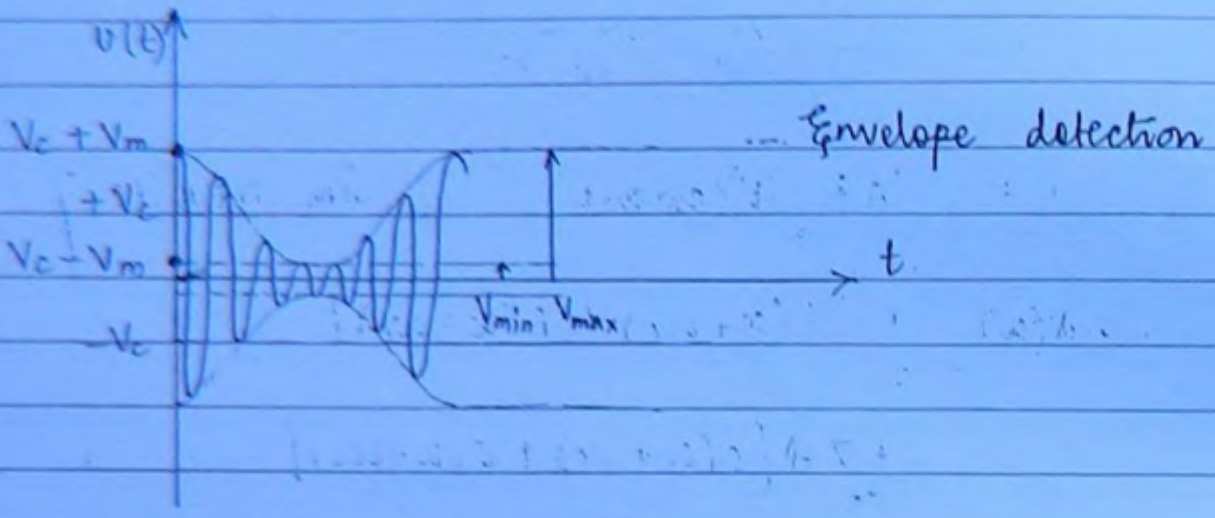
$$V(\omega) = \frac{1}{2}[F(\omega+\omega_c) + F(\omega-\omega_c)]$$

$$+ \pi A[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$



Graphical Representation of AM - DSB/FC

$$v(t) = V_c(1 + m_a \cos \omega_m t) \cos \omega_c t$$



... Envelope detection

$$V_{min} = V_c - V_m$$

$$V_{max} = V_c + V_m$$

$$m_a = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{V_m}{V_c}$$

If $m_a = 1$

$V_{min} = 0$

$V_m = V_c$

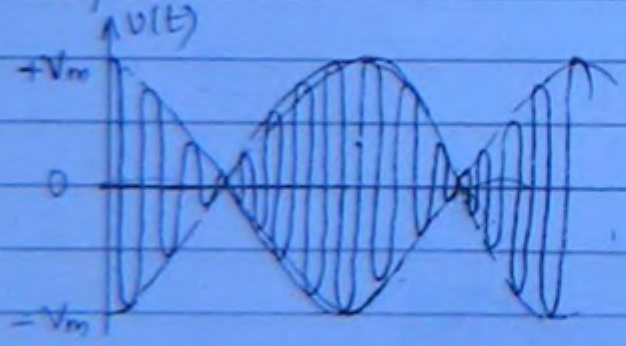
If $m_a = 0$

$V_m = 0$

$V_{max} = V_{min}$

unmodulated carrier

AM - DSB/SC



... Coherent or synchronous detection

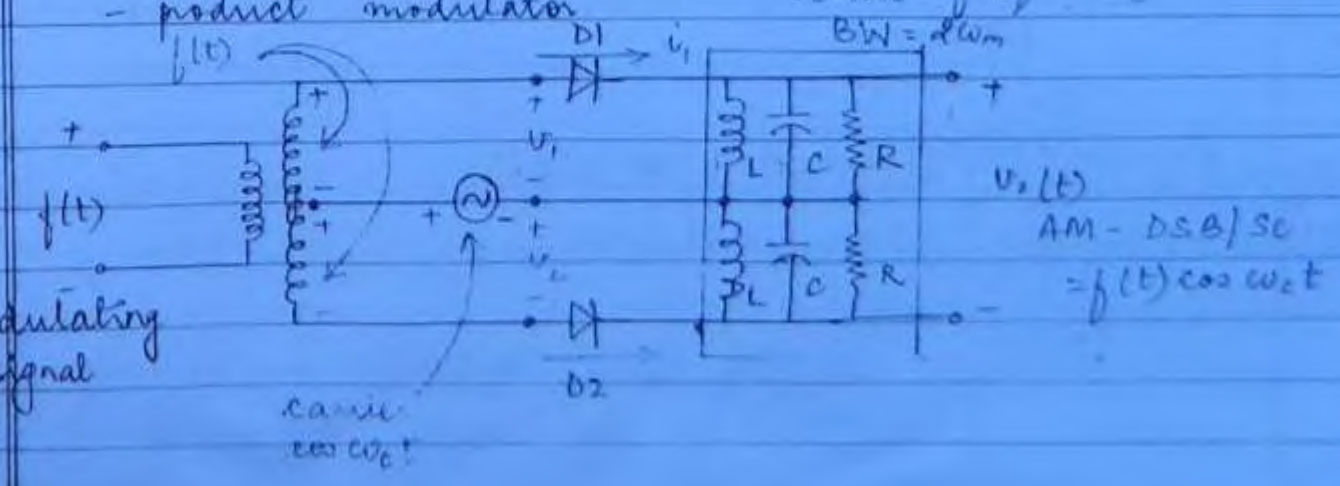
1. **Modulation Index** is the measure of variation of the carrier voltage with the modulating signal. Higher is the variation of the carrier with the modulating signal, higher is the value of modulation index & vice-versa. (15)

2. The original modulating ~~the~~ signal can be recovered from the standard AM signal by using **Envelope detection**. This envelope detector has simplest circuitry & ∴ can be used for broadcast purpose.

3. For suppressed carrier signal original modulating signal can be recovered by using **Coherent Detection** (or) **Synchronous Detection**. This system has complex circuitry & ∴ cannot be used for broadcast purpose.

Ex Using a circuit diagram explain the method for the generation of AM-DSB/SC signal using square law diodes.

Sol Generation of AM - DSB/SC
- square law modulator
- balanced modulator
- product modulator



$i_1 = aV_1 + bV_1^2$ ... D1

$i_2 = aV_2 + bV_2^2$ ... D2

$V_0 = (i_1 - i_2) R$

(16)

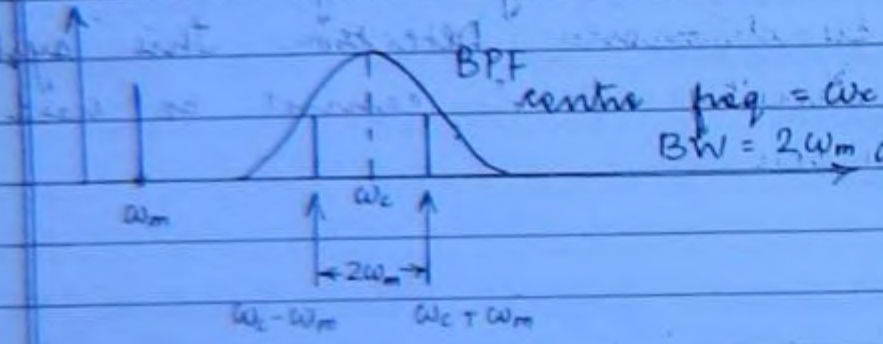
$V_1 = \cos \omega_c t + f(t)$

$V_2 = \cos \omega_c t - f(t)$

$V_0 = aR[\cos \omega_c t + f(t)] + bR[\cos^2 \omega_c t + f^2(t) + 2 \cos \omega_c t f(t)]$
$- aR[\cos \omega_c t - f(t)] - bR[\cos^2 \omega_c t + f^2(t) - 2 \cos \omega_c t f(t)]$

$V_0 = 2aR f(t) + 4bR f(t) \cos \omega_c t$

amplitude ratio = ω_m ... $\omega = \omega_c \pm \omega_m$

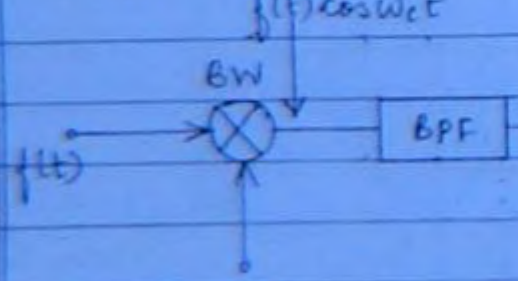


Output of BPF

$V_0(t) = k f(t) \cos \omega_c t$ ... AM - DSB/SC

where

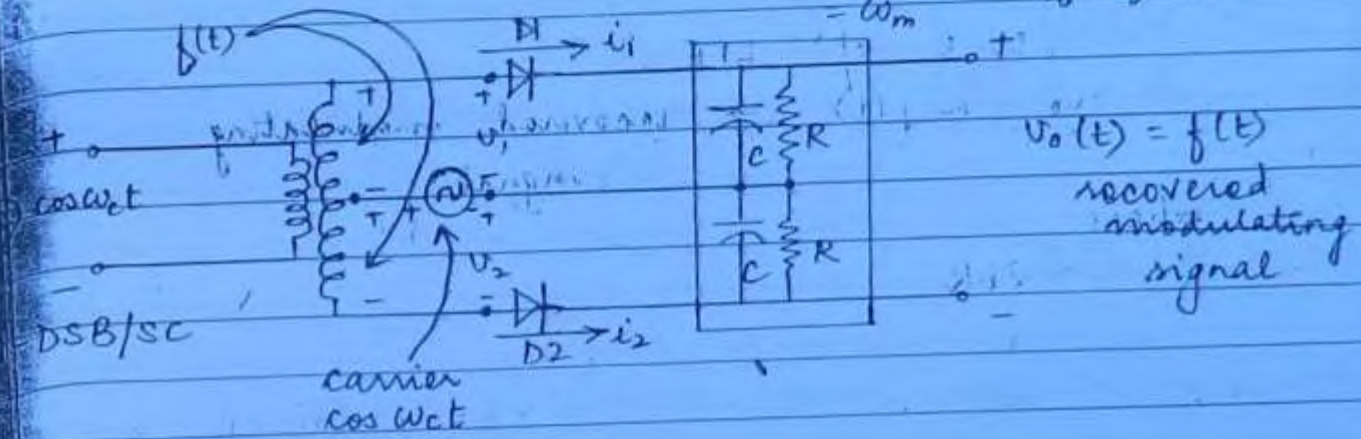
$k = 4bR$



Using necessary mathematical steps explain how the original modulating signal can be regenerated from AM - DSB/SC using a synchronous / coherent detector. Hence explain the significance of the pilot carrier. (17)

Detection of AM - DSB/SC
- square law demodulator
- synchronous detector
- coherent detector

Each is a LPF with cut-off freq $= \omega_m$



Analysis

$$V_0 = (i_1 - i_2) R$$

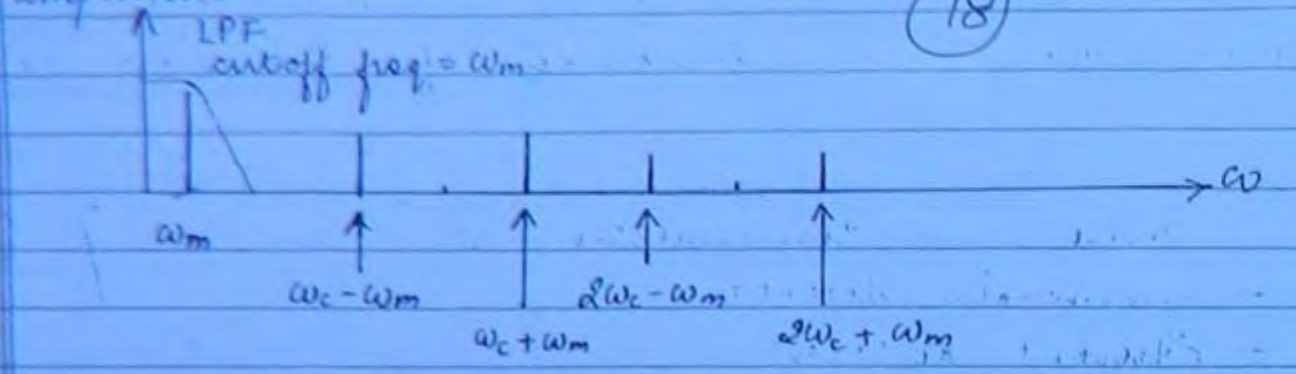
$$i_1 = a v_1 + b v_1^2 \quad \text{... D1}$$
$$i_2 = a v_2 + b v_2^2 \quad \text{... D2}$$

$$v_1 = \cos \omega_c t + f(t) \cos \omega_c t$$
$$v_2 = \cos \omega_c t - f(t) \cos \omega_c t$$

$$V_0 = aR[\cos \omega_c t + f(t) \cos \omega_c t] + bR[\cos^2 \omega_c t + f^2(t) + 2 \cos^2 \omega_c t f(t)]$$
$$- aR[\cos \omega_c t - f(t) \cos \omega_c t] - bR[\cos^2 \omega_c t + f^2(t) - 2 \cos^2 \omega_c t f(t)]$$

$$V_0 = 2aR f(t) \cos \omega_c t + 4bR f(t) \underbrace{\cos^2 \omega_c t}_{= \frac{1}{2}(1 + \cos 2\omega_c t)}$$

$$v_o = \underbrace{2aR f(t) \cos \omega_c t}_{=\omega_c \pm \omega_m} + \underbrace{2bR f(t)}_{=\omega_m} + \underbrace{2bR f(t) \cos 2\omega_c t}_{=2\omega_c \pm \omega_m}$$



output of LPF

$v_o = K f(t)$ = recovered modulating signal

where

$K = 2bR$

* This circuit has more complexity since a local carrier has to be generated having same magnitude & same frequency as that of the carrier at the transmitting end.

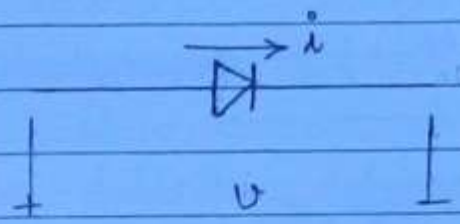
* Instead of suppressing the carrier 100% we actually suppress it by 95% & 5% of the carrier is transmitted along with the 2 side bands this small 5% of the carrier is called the **Pilot carrier** & is used to generate the local carrier at the demodulator end.

* At the recieving end this carrier is amplified & is passed through a PLL circuit so that a local carrier is generated having same magnitude & phase as that of the carrier at

the transmitting end. Therefore the carrier at the transmitting end & at the recieving end are synchronised or in coherence.

* The circuit associated to obtain the local carrier is much more complex & therefore such system cannot be used for broadcast.

Diode as a square law device -



$$i = I_0 \left(e^{v/(\eta V_T)} - 1 \right) \quad \text{... diode equation}$$

$\eta = 1$ ... Ge
$\eta = 2$ ... Si

$$V_T = \frac{kT}{e} = \frac{T}{11600} \cong 26\ mV \quad \text{at } T = 300^\circ K$$

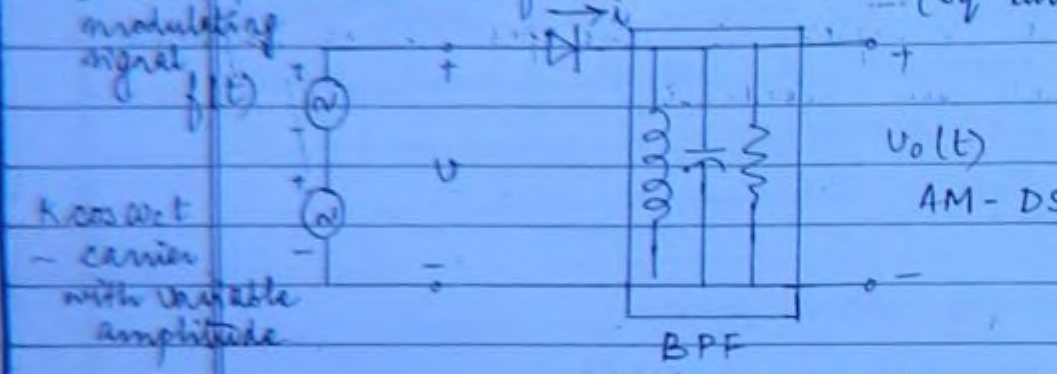
$$\Rightarrow i = I_0 \left(1 + \frac{v}{\eta V_T} \cdot \frac{1}{1!} + \left(\frac{v}{\eta V_T} \right)^2 \frac{1}{2!} + \dots \right) \quad \text{--- 1)}$$

$$\Rightarrow \boxed{i \cong av + bv^2} \quad \dots \quad a = \frac{I_0}{\eta V_T}$$

$$b = \frac{I_0}{2(\eta V_T)^2}$$

Ex: Using necessary mathematical steps explain the working of a square law modulator for the generation of a standard AM signal. (20)

Sol: Generation of AM-DSB/FC (sq law modulator)



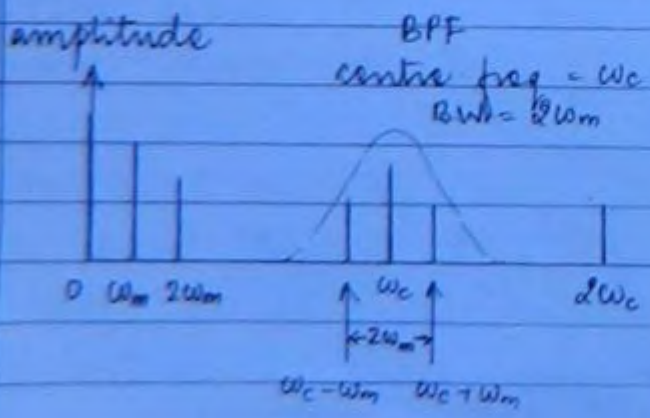
$V_0 = iR$

$i = av + bv^2$

$v = f(t) + k\cos\omega_c t$

$V_0 = aR[f(t) + k\cos\omega_c t] + bR[f^2(t) + k^2\underbrace{\cos^2\omega_c t}_{=\frac{1}{2}[1+\cos 2\omega_c t]} + 2kf(t)\cos\omega_c t]$

$V_0 = \underbrace{aRf(t)}_{=\omega_m} + \boxed{akR\cos\omega_c t}_{=\omega_c} + \underbrace{bRf^2(t)}_{=2\omega_m} + \underbrace{\frac{1}{2}k^2bR}_{=0} + \underbrace{\frac{1}{2}k^2bR\cos 2\omega_c t}_{=2\omega_c} + \boxed{\underbrace{2kbRf(t)\cos\omega_c t}_{=\omega_c \pm \omega_m}}$



Output of BPF

$V_o = \underbrace{k_1 \cos \omega_c t}_{\text{free carrier}} + \underbrace{k_2 f(t) \cos \omega_c t}_{\text{AM-DSB/SC}}$

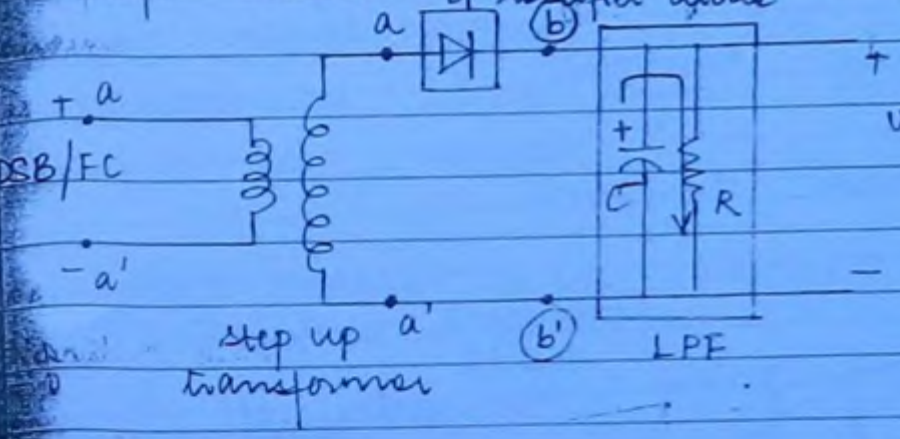
AM - DSB/FC

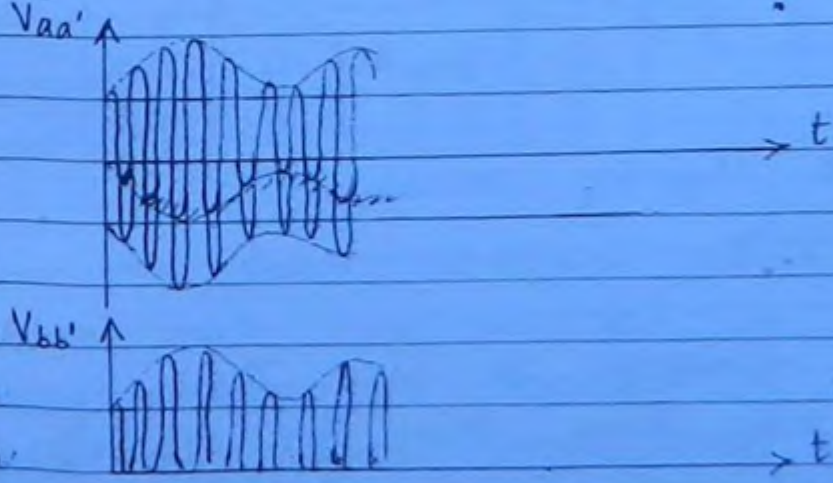
$k_1 = ak_R$

$k_2 = 2bk_R$

Explain the working of an envelope detector for the recovery of original modulating signal from the standard AM signal. How will you select the RC constant of the LPF in the envelope detector.

Envelope detector (demodulation of AM - DSB/FC)





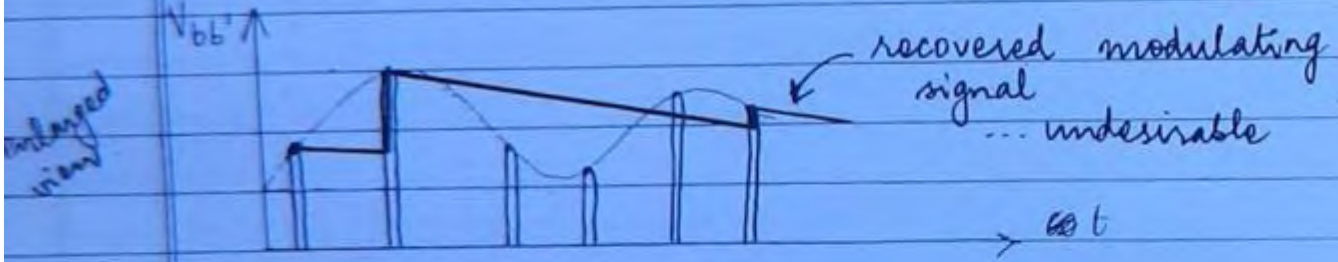
1. During +ve half of carrier D1 ... FB
C charges with time constant $R_f C$ ... small
(R_f → forward resistance of diode)

2. During -ve half of carrier D2 ... RB
C discharges through R with time constant = RC

* The RC constant must be selected in such a manner so that as far as possible the capacitor voltage must follow the envelope of the composite signal to recover the original modulating signal.

Choice of RC time constant

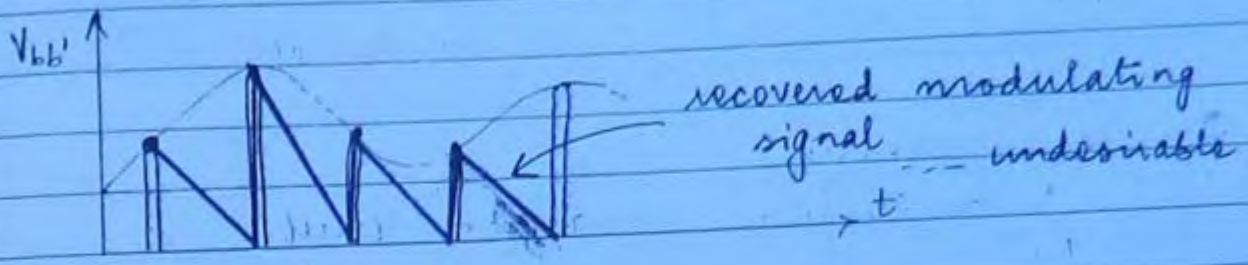
Case 1 : High time constant



... negative peak clipping occurs
... undesirable

When the RC time constant is high, the capacitor discharges very slow, then the capacitor voltage does not follow the envelope of the composite signal & the negative peak clipping of the signal occurs. ∴ high RC time constant is undesirable since the original modulating signal

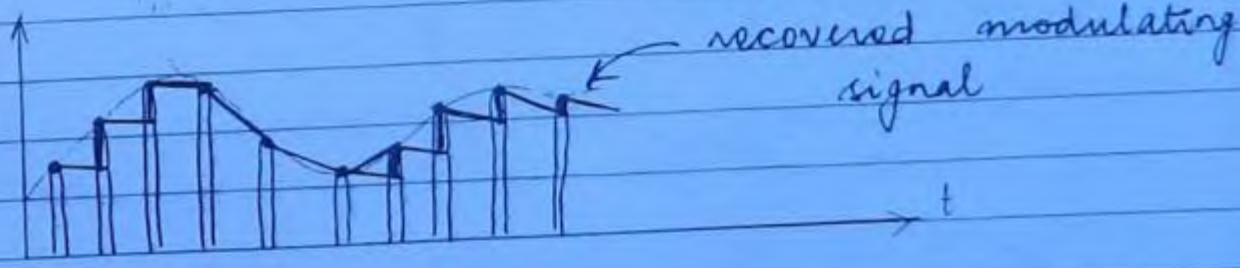
case d: Low time constant



-- diagonal clipping occurs
-- undesirable

When the RC time constant is small, the capacitor discharges very fast. Then the capacitor voltage does not follow the envelope of the composite signal & diagonal clipping of the signal occurs. i.e. low RC time constant is undesirable since original modulating signal cannot be recovered

case 3: Medium time constant



The RC time constant should have medium value such that as far as possible the capacitor voltage follows the envelope of the composite signal then the original modulating signal is recovered with minimum distortion or minimum noise
This time constant depends upon

i) Modulation Index
ii) Maximum frequency component of modulating signal

$$\frac{1}{RC} \geq \frac{\omega_m \cdot m_a}{\sqrt{1-m_a^2}}$$ ... exact result ... to be derived

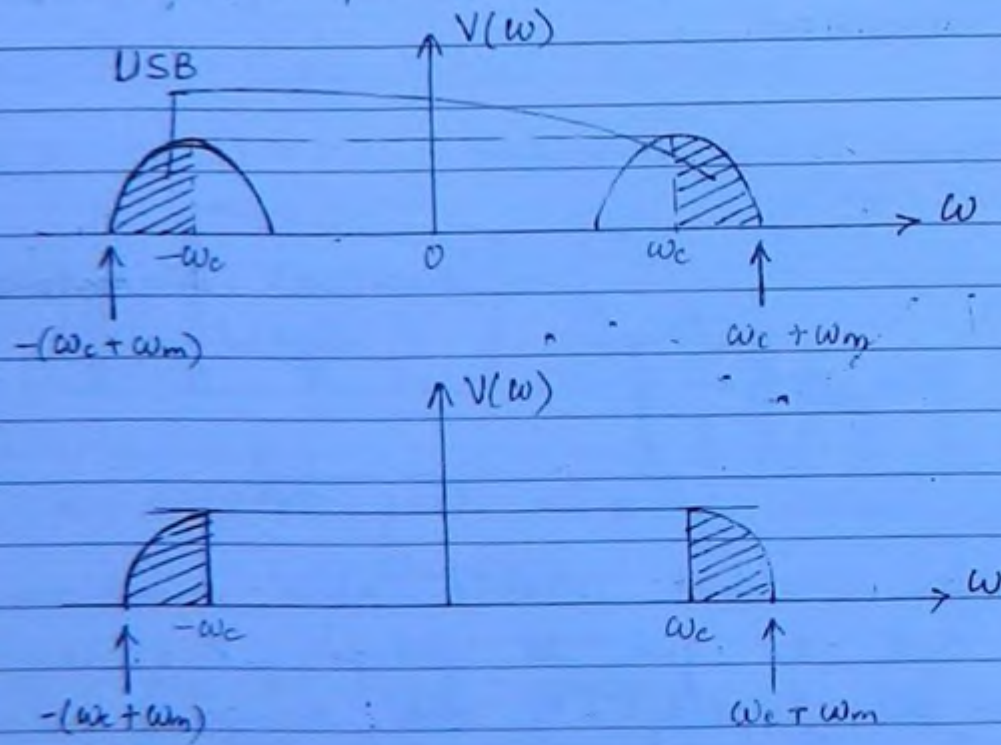
If m_a is small

$$\frac{1}{RC} \geq \omega_m \cdot m_a$$ ... approx result

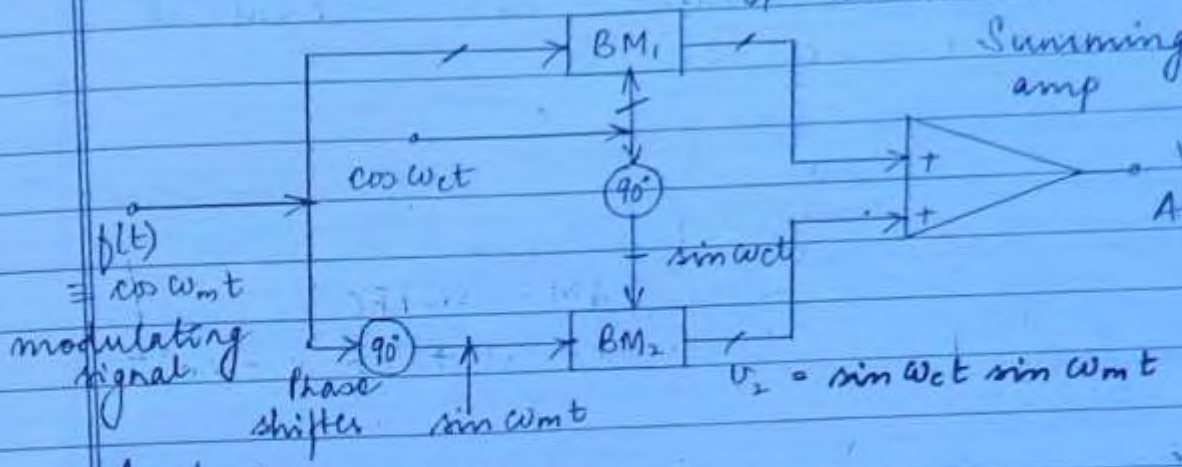
Using necessary mathematical steps explain the phase shift method for the generation of AM-SSB/SC signal. For the generation of SSB signals explain why is the filter method is not preferred.

In General of SSB signals

AM-SSB/SC



Phase Shift Method for the generation of AM:- SSB/SC signal



Analysis -

$$V_o = V_1 + V_2$$

$$= \cos \omega_c t \cos \omega_m t + \sin \omega_c t \sin \omega_m t$$

$$= \frac{1}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$+ \frac{1}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

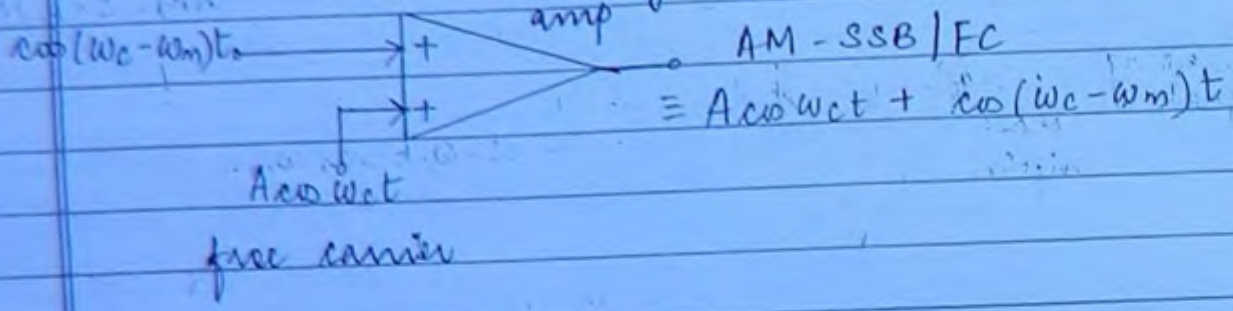
$$V_o = \cos(\omega_c - \omega_m)t \quad \text{... AM SSB/SC (USB is suppressed)}$$

* For the generation of SSB signals filter method is not preferred since any practical filter does not have sharp cut-off characteristics. ∴ such filter has gradual cut-off characteristics & ∴ along with one sideband a part of other sideband is also generated.

* Therefore SSB signal is generated by electrically neutralising one side band by providing a 90° phase shift to the carrier & to the modulating signal

* To obtain a phase shift of exactly 90°, its thermal characteristics and stabilised by putting the phase shifter in a constant temperature chamber.

Generation of AM-SSB/FC



Detection of:

AM-DSB/FC, AM-SSB/FC → using envelope detection

AM-DSB/SC, AM-SSB/SC → using synchronous or coherent detection

ANGLE MODULATION –

- PM — Phase mod
- FM — Freq mod

} f_c : 88 MHz – 108 MHz ... broadcast range

PM

In phase modulation, phase of the carrier is varied in accordance with the instantaneous value of the amplitude of the modulating signal keeping the amplitude & the frequency of the carrier fixed.

$f(t)$ ... modulating signal

$v_c(t) = A \cos \omega_c t = A \cos \theta(t)$ ... carrier signal

$\theta = \omega_c t$

Modulated signal

$v_{PM}(t) = A \cos \theta_i(t)$

where $\theta_i = \theta + (k_p . f(t))$

$= \omega_c t + k_p . f(t)$

$$\boxed{v_{PM}(t) = A \cos [\omega_c t + k_p f(t)]}$$ ... general expression of PM signal

If $f(t) = v_m(t) = V_m \cos \omega_m t$

$$\boxed{v_{PM}(t) = A \cos [\omega_c t + m_p \cos \omega_m t]}$$ PM signal

$m_p = K_p V_m$

modulation index

FM

In frequency modulation, frequency of the carrier is varied in accordance with the instantaneous value of the amplitude of the modulating signal keeping the amplitude & phase of the carrier fixed

$f(t)$ ... modulating signal

$v_c(t) = A \cos \omega_c t = A \cos \theta(t)$

$\omega_i = \omega_c + (k_f . f(t))$

$$\theta_i = \int_0^t \omega_i \, dt = \int_0^t [\omega_c + k_f f(t)] \, dt$$

$$\theta_i = \omega_c t + k_f \int_0^t f(t) \, dt$$

Modulated signal

$$v_{FM}(t) = A \cos \theta_i(t)$$

$$\boxed{v_{FM}(t) = A \cos \left[\omega_c t + k_f \int_0^t f(t) \, dt \right]}$$: general expression for FM signal

If $f(t) = V_m \cos \omega_m t$

$$\boxed{v_{FM}(t) = A \cos [\omega_c t + m_f \sin \omega_m t]}$$ FM signal

The PM & FM signals are interdependent, since any variation in the frequency will always cause some phase variation & vice versa. ∴ we cannot have a pure PM signal or a pure FM signal.

If we integrate the modulating signal & the carrier is allowed for phase modulation the result is FM signal. This method represents an indirect method for the generation of FM signal.

If we differentiate the modulating signal & allow the carrier for FM the result is a PM signal. This method is never used since amount of noise increases in the system due to differentiation.

Summary

$f(t)$ ... modulating signal

↓

Integrate it

$$g_1(t) = \int_0^t f(t)\,dt$$

↓

Phase modulate the carrier using $g_1(t)$

↓

NBFM signal ... Indirect method of generation of FM.

.... Armstrong's method

$f(t)$ ... modulating signal

↓

$$g_2(t) = \frac{df(t)}{dt}$$

↓

freq modulate the carrier using $g_2(t)$

↓

PM signal ... never used

.... noise in the system increases

Power required for FM signal

(30)

$$v_{FM}(t) = A \cos\left[\omega_c t + K_f \int_0^t f(t)\,dt\right]$$

$$= A \cos\left[\omega_c t + m_f \sin(\omega_m t)\right]$$

$$P = \overline{v_{FM}^2(t)} = A^2 \cdot \underbrace{\overline{\cos^2[\ldots]}}_{=1/2}$$

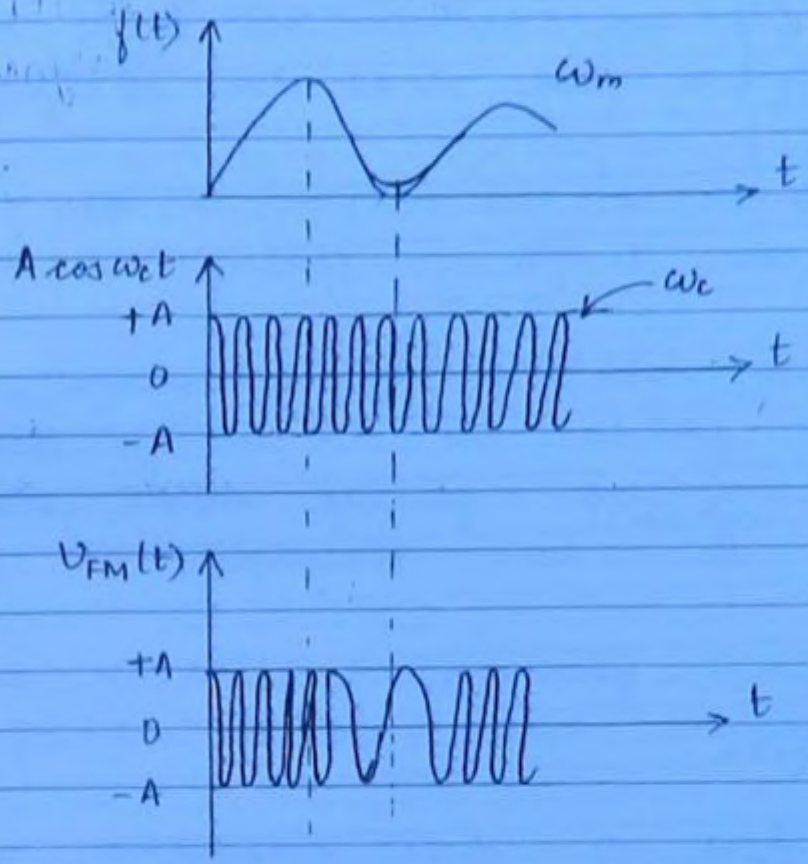
$$P = \frac{1}{2}A^2 \quad \ldots \text{FM}$$

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \quad \ldots \text{AM}$$

The total power in the FM & PM signal always remains constant irrespective of the value of modulation index. This power depends only upon the amplitude of the carrier.

For AM signal the total power transmitted depends upon the value of modulation index in addition to carrier amplitude.

case-1 in time domain



modulating signal ; ω_m

high freq carrier $\omega_c >> \omega_m$

modulated signal

case-2 in frequency domain

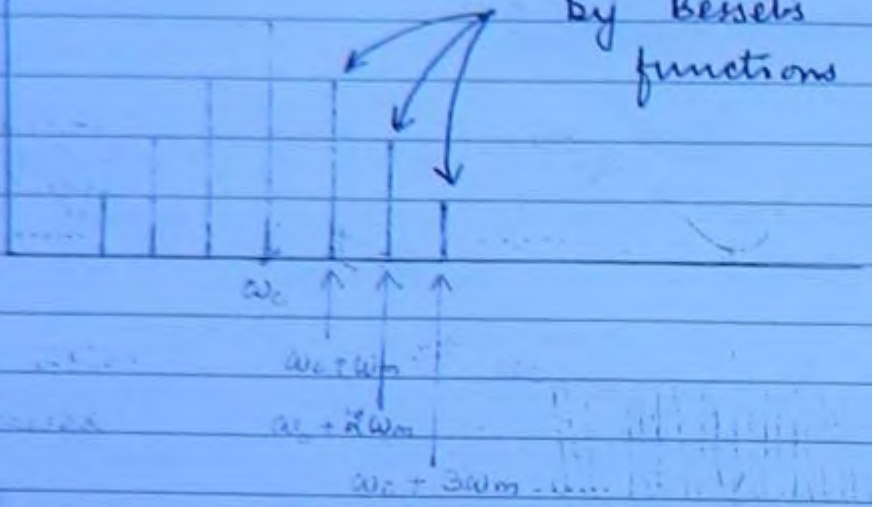
$$V_{FM}(t) = A \cos[\omega_c t + m_f \sin \omega_m t]$$

$\cos \omega_c t$

$\cos[m_f \sin \omega_m t]$, contains infinite no of terms & magnitude of each terms is controlled by Bessel's function

⇒ Infinite number of SB

amplitude of each freq comp amp. is controlled by Bessel's functions



Parameters of FM signal

1. ω_m ; f_m
2. m_f
3. δ freq deviation

$$\delta = m_f \cdot \omega_m \quad \text{rad/sec}$$
$$\delta = m_f \cdot f_m \quad \text{Hz}$$

Any FM signal requires infinite amount of BW for transmission, since the number of side bands are infinite in FM signal.

As the frequency of the side band increases its amplitude decreases Then we consider first few pair of significant side bands which contain almost 98% of total power & neglecting those higher side bands which contribute insignificant amount of power. Then amount of BW

required for the transmission of FM signal is finite without affecting quality of the voice signal.

* The total BW required depends upon number of significant side bands and is given by Carson's rule

* FM requires higher BW as compared to AM system but the quality of signal becomes better.

* FM system is hi-fi (high fidelity) system ∴ such system has the ability to reproduce a signal with same quality as it was transmitted at the modulator end.

* The modulating frequency f_m controls the separation between two successive side bands.

The modulation index m_f controls the number of significant side bands.

The frequency deviation δ controls total BW required for the transmission of FM signal.

Since FM signal is a constant amplitude signal, the signal to noise ratio for such system is much higher as compared to that of AM system.

Types of FM systems –

case 1 : WBFM .. wide band FM

$m_f >> 1$

$BW \cong 2(\delta + \omega_m)$ .... rad/sec

.... Carson's Rule

.... used in FM broadcast

$\cong 2(m_f \omega_m + \omega_m)$

$\cong 2\omega_m(m_f + 1)$

$BW \cong 2\omega_m m_f \cong 2\delta$ $\delta_{max} = 75$ KHz

$f_c \pm \delta$

case 2 : NBFM .. narrow band FM

$m_f << 1$ .... used in mobile radio

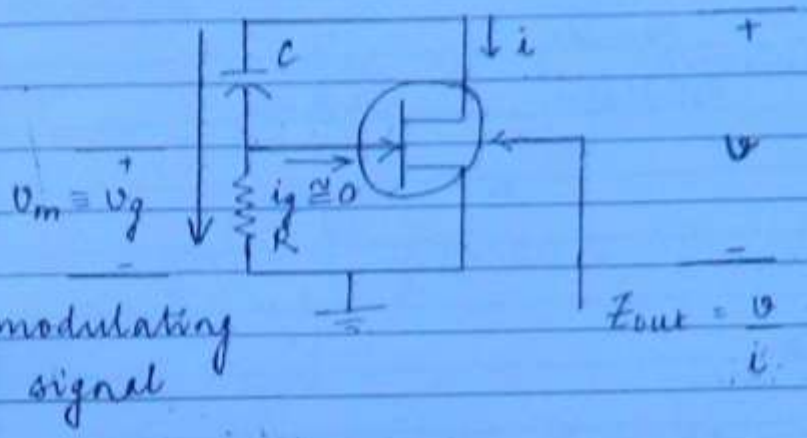
$BW \cong 2\omega_m(m_f + 1)$

$BW \cong 2\omega_m$ .... same as AM - DSB system

.... contains one pair of SB

Generation of WBFM signal -

Principle Reactance mod



$$Z_{out} = \frac{v}{i} \quad \text{..... (1)}$$

$$g_m = \frac{i}{v_g} \quad \text{..... (2)}$$

$$v_g = \frac{R}{R + \frac{1}{j\omega C}} \cdot v = \frac{Rv}{R - jX_c} \quad \text{..... (3)}$$

$$g_m = \frac{i(R - jX_c)}{Rv}$$

$R << X_c$

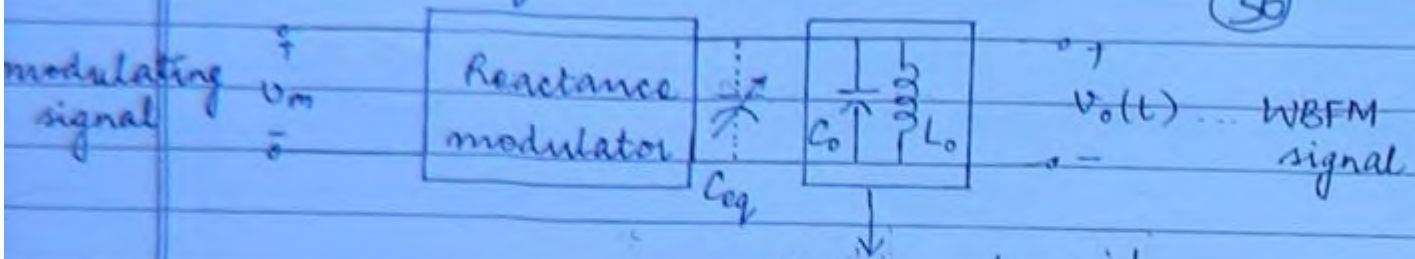
$$\frac{v}{i} = Z_{out} = \frac{R - jX_c}{g_m R} \cong \frac{1}{+j g_m R \omega C}$$

$$\cong \frac{1}{j\omega(g_m RC)} \cong \frac{1}{j\omega C_{eq}}$$

reactive impedance

$$C_{eq} = g_m RC$$

Generation of WBFM



V_m varies

↓

$g_m = \frac{i}{V_m}$ varies

↓

$C_{eq} = g_m RC$ varies

↓

$C = C_o + C_{eq}$ .... varies

↓

$f_c' = \frac{1}{2\pi\sqrt{L_o C}} = \frac{1}{2\pi\sqrt{L_o(C_o + C_{eq})}}$ varies

↓

WBFM signal

Hence the frequency at the output varies in accordance with the instantaneous value of the amplitude of incoming modulating signal and therefore this results in FM signal.

Since there is no restriction on the dynamic range of the input signal then the variation in frequency of the signal at the output of the modulator is also large resulting in WBFM signal

Generation of NBFM signal -

... Indirect method

... Armstrong's method

Principle

$f(t)$ ... modulating signal

↓

integrate it

$$g(t) = \int_0^t f(t)\, dt$$

↓

phase modulate the carrier using $g(t)$

↓

NBFM signal

PM

option - 1

fix the carrier

↓

Provide phase shift of 90° to the 2-SB

↓

PM signal

.... not used

option - 2

fix the 2-SB

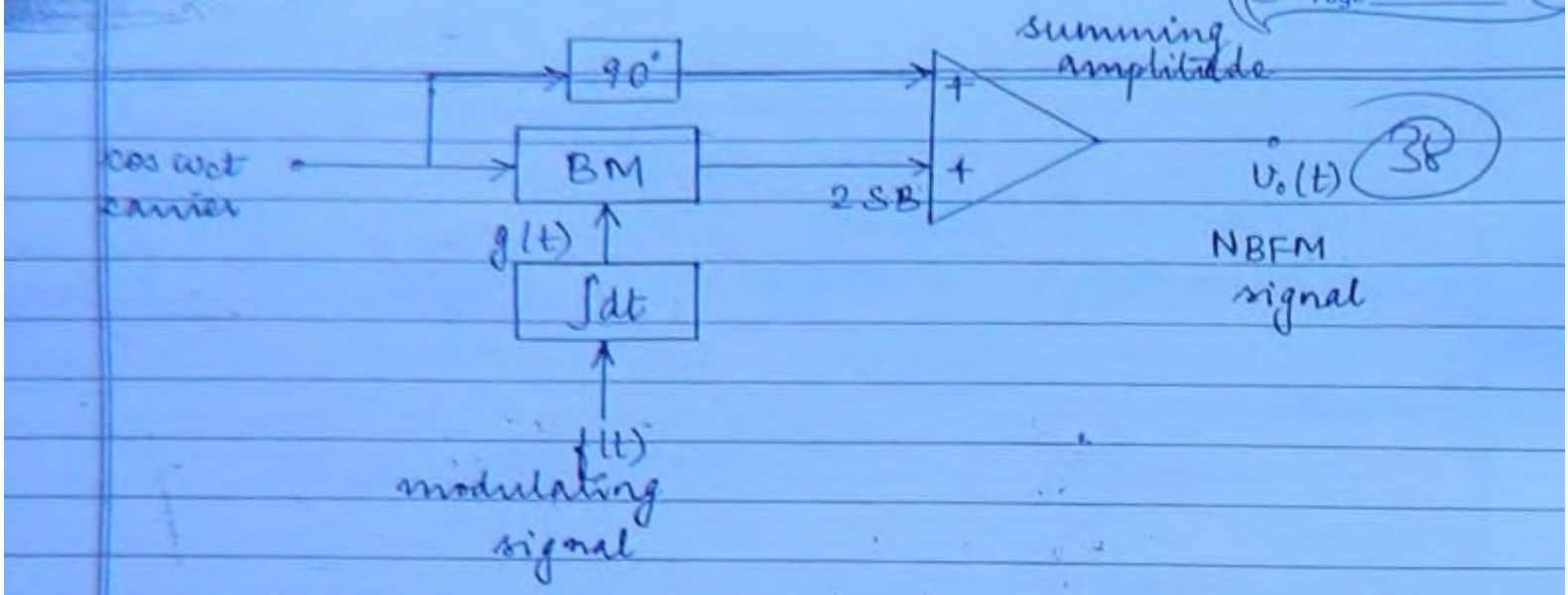
↓

Provide phase shift of 90° to the carrier

↓

PM signal

... always preferred



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* The circuit required for the generation of NBFM signal is more complex as compared to generation of WBFM signal.

* If the NBFM signal is available then the WBFM signal can be generated by passing the NBFM signal through a series of frequency multipliers which are essentially frequency doublers or frequency triplers.

Pre-emphasis & de-emphasis
.. used only in FM systems

audio signal → fundamental + large no. of harmonics

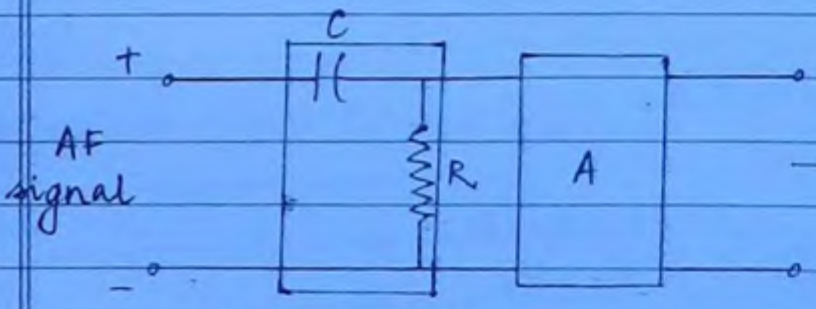
$f \uparrow \rightarrow$ amplitude $\downarrow$
$\rightarrow S \downarrow$
$\rightarrow \left(\frac{S}{N}\right) \downarrow$

* Therefore, the high frequency components which have low SNR are boosted or emphasised prior to transmission of signal.
* This is done by using a pre-emphasis circuit if is used at the transmitting end just before modulation takes place.

* Since the relative SNR for various frequency components has been disturbed, ~~than~~ then those frequency components which were initially boosted are now brought down to the same level to keep the same quality of signal.

* This is done by using a de-emphasis circuit if is used at the recieving end just after demodulation takes place.

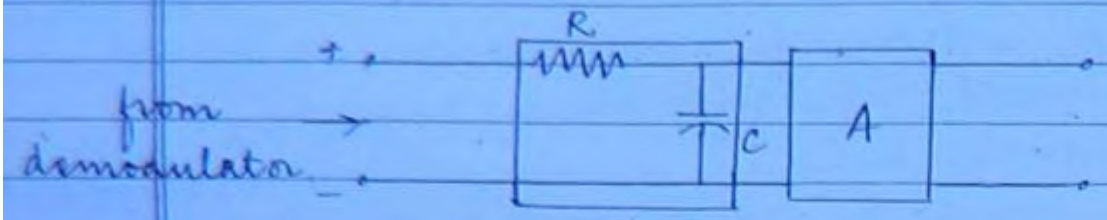
* The RC time constant of pre-emphasis & de-emphasis circuit is kept same of the order of ≈ 75 μsec.



$T = RC \approx 75\ \mu sec$

PRE EMPHASIS CIRCUIT

...used at transmitting end before modulator



$T = RC$

$\simeq 75\ \mu sec$

... used at the receiving end just after demodulator

Ex AM signal is given by :-

$$v_{AM}(t) = 0.1\,[1 + 0.5 \cos 6280\, t] \cdot \sin(10^7 t + 45^\circ)$$

Find all the parameters for this AM signal

Sol.

$$v_{AM}(t) = V_c (1 + m_a \cos \omega_m t) \sin(\omega_c t + \phi)$$

1. AM - DSB / FC

Carrier -

2. Amplitude $V_c = 0.1\ V$
3. Phase $\phi = 45^\circ$
4. Freq $\omega_c = 10^7 = 10\ M$ rad/sec

$$f_c = \frac{10^7}{2\pi} = 1590\ kHz$$

5. $v_c(t) = V_c \sin(\omega_c t + \phi)$

$= 0.1 \sin(10^7 t + 45^\circ)$

6. $m_a = 0.5 = 50\%$

7.

Modulating signal:

7. $\omega_m = 6280$ rad/sec.

$f_m = \frac{6280}{2\pi} = 1$ KHz.

8. amplitude

$m_a = \frac{V_m}{V_c} \Rightarrow V_m = m_a V_c$

$= 0.5 \times 0.1$

$= 0.05$ V

9. $v_m(t) = V_m \cos \omega_m t$

$= 0.05 \cos 6280 t$

10. SB freq $= f_c \pm f_m$

$= (1590 \pm 1)$ kHz

$= \begin{cases} 1591 \text{ kHz} \\ 1589 \text{ kHz} \end{cases}$

11. SB ampl $= \frac{1}{2} m_a V_c = \frac{1}{2} \times 0.5 \times 0.1 = 0.025$ V

12. Transmission efficiency

$\eta = \frac{m_a^2}{2 + m_a^2} \times 100 = \frac{0.5^2}{2 + 0.5^2} \times 100 \cong 11.1\%$

13. $P_t = P_c \left(1 + \frac{m_a^2}{2}\right)$

$P_c = \frac{V_c^2}{2R}$ V_c is peak power so it's divided by $\sqrt{2}$ for RMS value

$= \frac{1}{2} \frac{V_c^2}{1} = \frac{1}{2}(0.1)^2$ W

$P_t = \frac{0.1^2}{2}\left(1 + \frac{0.5^2}{2}\right)$ W

14. Power in each SB

$P_{1\text{-SB}} = P_c \frac{m_a^2}{4}$ W

Ex A carrier is amplitude modulated by a combination of 2 sinusoidal modulating signals with individual modulation indicies of 0.3 & 0.4 respectively. Calculate power in SB as a % of total power in the modulated signal.

Sol
$$m_{a_1} = 0.3$$
$$m_{a_2} = 0.4$$
$$m_a = \sqrt{m_{a_1}^2 + m_{a_2}^2} = \sqrt{0.3^2 + 0.4^2} = 0.5$$

$$\frac{P_{SB}}{P_t} \times 100 = \eta = \frac{m_a^2}{2 + m_a^2} \times 100$$
$$\cong 11.1\ \%$$

Ex A 5 KW SSB-SC signal is to be replaced, by a standard AM signal with same power content. Find:
i) carrier power P_c
ii) power in each SB for 90% modulation index

Sol
$$P_{AM-SSB/SC} = 5\ KW$$
$$= P_{AM-DSB/FC}$$
$$\Rightarrow P_c\left(1 + \frac{m_a^2}{2}\right) = 5\ KW$$
$$\Rightarrow P_c\left(1 + \frac{0.9^2}{2}\right) = 5\ KW$$
$$\Rightarrow P_c = 3.56\ KW$$

$$P_{1-SB} = P_c \frac{m_a^2}{4} = 0.72\ KW$$

also each SB $= \frac{P_c}{2}$

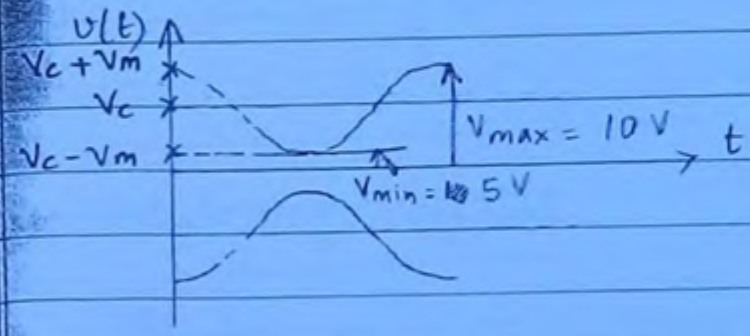
A 5 MHz RF waveform is modulated with 3 kHz signal. The carrier maximum peak-to-peak v/g is 20 V & minimum peak-to-peak v/g is 10 V. Find:

(43)

i) unmodulated carrier v/g
ii) modulation index
iii) side band frequencies
iv) carrier power
v) total power

Assume a load resistance of 200 ohms.

$f_c = 5\ MHz$
$f_m = 3\ kHz$
$R = 200\ \Omega$



$V_{max} = V_c + V_m = 10$

$V_{min} = V_c - V_m = 5$

i) $V_c = 7.5$

$V_m = 2.5$

ii) $m_a = \frac{V_m}{V_c} = \frac{2.5}{7.5} = \frac{1}{3} = 0.33 = 33\%$

iii) SB freq $= f_c \pm f_m$

$= (5 \pm 0.003)\ MHz$ — 5.003 MHz, 4.997 MHz

iv) $P_c = \frac{V_c^2}{2R_L} = 0.141\ W$

v) $P_t = P_c \left(1 + \frac{m_a^2}{2}\right) =$

$= 0.149\ W$

Ex RMS antenna current is 5 A when modulated to 50% level. Find new value of RMS antenna current if modulation index is increased to 75% level. (44)

Sol Case I

$$I_t = I_c \sqrt{1 + \frac{m_a^2}{2}}$$

$$5 = I_c \sqrt{1 + \frac{0.5^2}{2}}$$

$$I_c = 4.72 \text{ A}$$

Case - II

$$I_t = I_c \sqrt{1 + \frac{m_a^2}{2}}$$

$$= 4.72 \sqrt{1 + \frac{0.75^2}{2}}$$

$$= 5.23 \text{ A}$$

Ex An FM signal is given by

$$V_{FM}(t) = 10 \sin (10^8 t + 15 \sin 2000t)$$

Find all the parameters related to this FM signal

Sol

$$V_{FM}(t) = A \sin (\omega_c t + m_f \sin \omega_m t)$$

$$= A \sin \left(\omega_c t + k_f \int_0^t f(t)\, dt\right)$$

carrier :-

1. $\omega_c = 10^8 = 100 \text{ M rad/sec}$

$$f_c = \frac{10^8}{2\pi} = 15.9 \text{ MHz}$$

2. ampl $A = 10\ V$

3. $V_c(t) = A \sin(\omega_c t)$
$= 10 \sin(10^8 t)$

4. $m_f = 15$

5. $m_f \gg 1$ WBFM

6. $\omega_m = 2000$ rad/sec
$f_m = \frac{2000}{2\pi} = 318\ Hz$

7. freq deviation $\delta = m_f \cdot f_m = (15 \times 318)\ Hz$

8. SB freq $= f_c \pm n f_m \quad ; \quad n = 1, 2, 3 \ldots$
$= 15.9\ MHz \pm n \times 318\ Hz$

9. $BW \cong 2(\delta + f_m) \ldots Hz$
$\cong 2 f_m (m_f + 1)$
$\cong 2 \times 318 (15 + 1) \ldots Hz$

** Modulating signal:

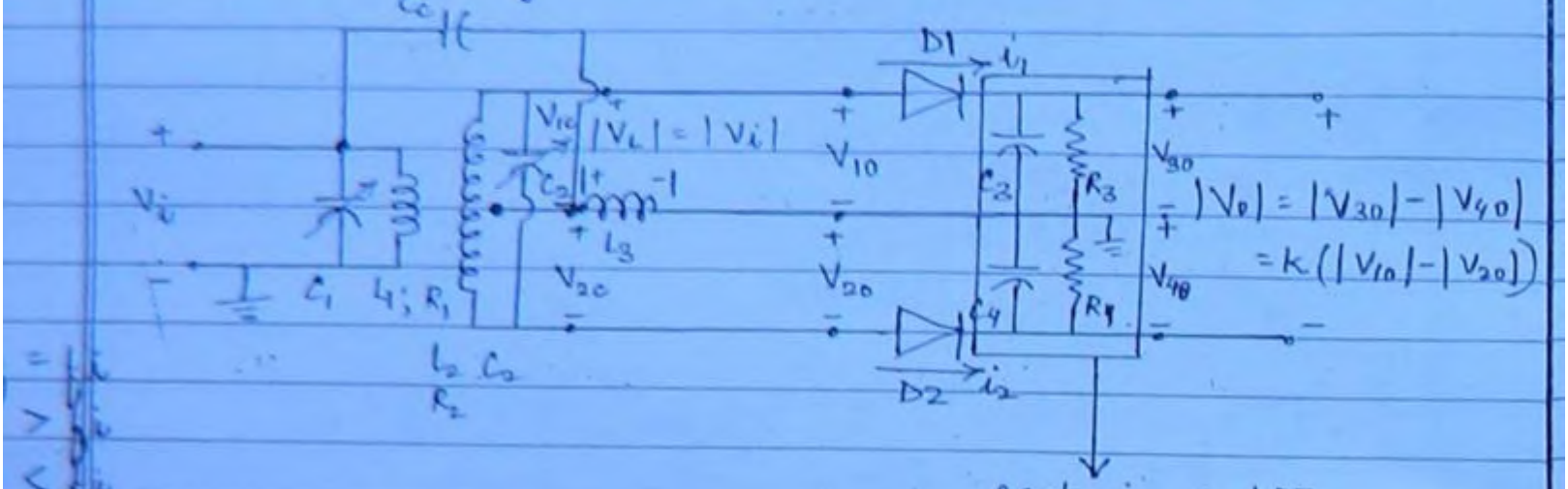
10. $K_f \int_0^t f(t)\, dt = 15 \sin 2000 t$

$f(t) = \underbrace{\frac{15 \times 2000}{K_f}} \cos 2000 t$

$= V_m \cos 2000 t$

FM Demodulator –

... Foster - Seeley Discriminator



= Intermediate frequency (IF) } – [10.7 MHz ... FM ←
455 kHz ... AM

1. during +ve half of input
   D1 conducts ; $|V_{30}|$ exists
   D2 OFF ; $|V_{40}| = 0$

$$|V_0| = |V_{30}|$$

2. during -ve half of input
   D1 OFF ; $|V_{30}| = 0$
   D2 conducts ; $|V_{40}|$ exists

$$|V_0| = |V_{40}|$$

3. during one complete input v/g cycle

$$|V_0| = |V_{30}| - |V_{40}|$$

$$|V_0| = k(|V_{10}| - |V_{20}|)$$

* The input of the network is a double tuned network & each is tuned at the centre frequency fi. The tuned circuit then behaves as a resistive, capacitive or inductive network depending upon whether incoming frequencies are equal to or more than or less than the centre frequency fi.

* The coupling capacitor Cc acts almost as a short circuit at the frequencies of operation

* The input tuned network & the inductor L3 are effectively connected in parallel & have equal voltages in magnitude but differ in phase. The phase difference may be 90° or more than 90° or less than 90° depending upon whether the incoming frequencies are equal to or more than or less than the centre frequency fi

Due to centre tapping V_{1c} & V_{2c} are equal in magnitude but are opposite in phase.

V_{10} is then phasor addition of the v/gs V_{1c} & V_L
Similarly V_{20} is the phasor addition of the v/gs V_{2c} & V_L

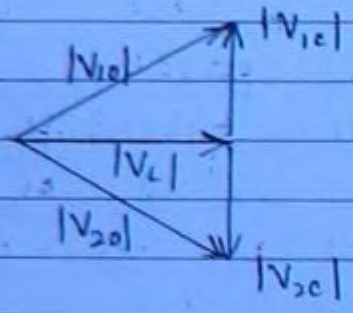
The output v/g V_o then depends upon the v/gs V_{10} & V_{20}

R3, C3 and R4, C4 represent LPFs and each has a cut-off frequency of ω_h

To find the output v/g variation with the variation in incoming frequencies w.r.t the centre frequency fi.

All the incoming frequencies are translated to the intermediate frequency so that incoming FM signal is always demodulated irrespective of the signal frequencies at the input of the radio receiver.

case 1 $f = f_i$

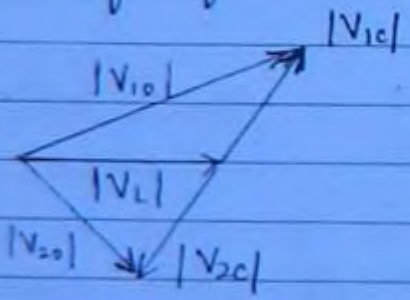


$|V_{10}| = |V_{20}|$

$|V_o| = k(|V_{10}| - |V_{20}|)$

$= 0$

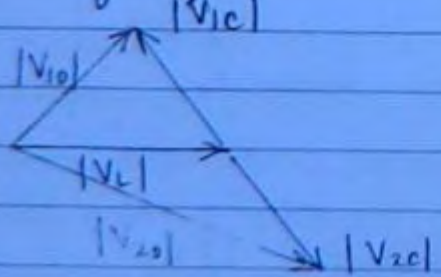
case 2 $f > f_i$



$|V_{10}| > |V_{20}|$

$|V_o| = +ve$

case 3 $f < f_i$

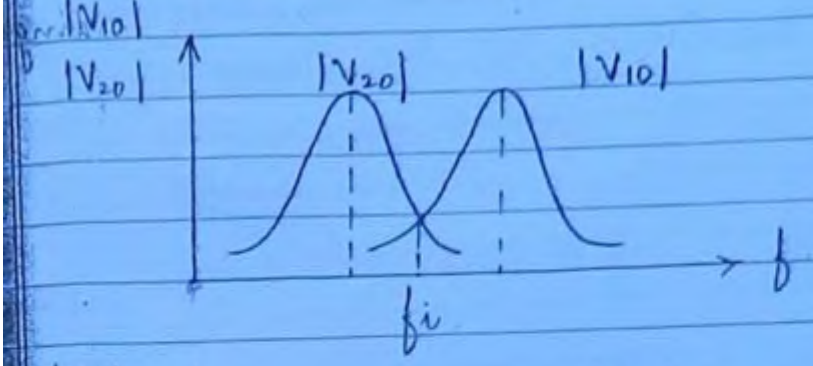


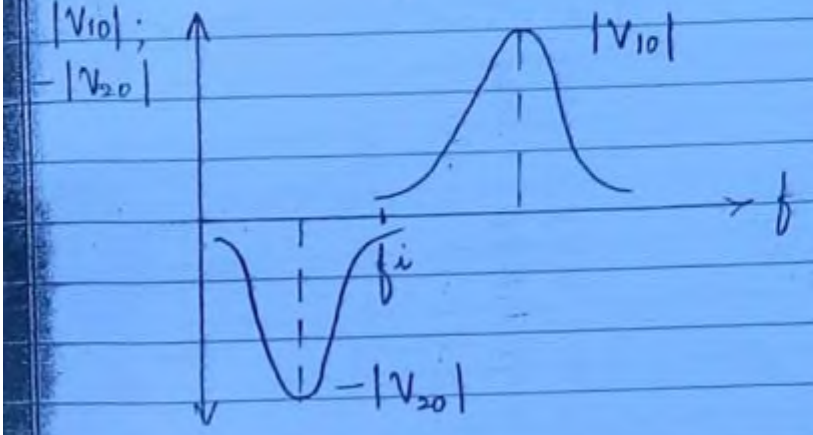
$|V_{10}| < |V_{20}|$

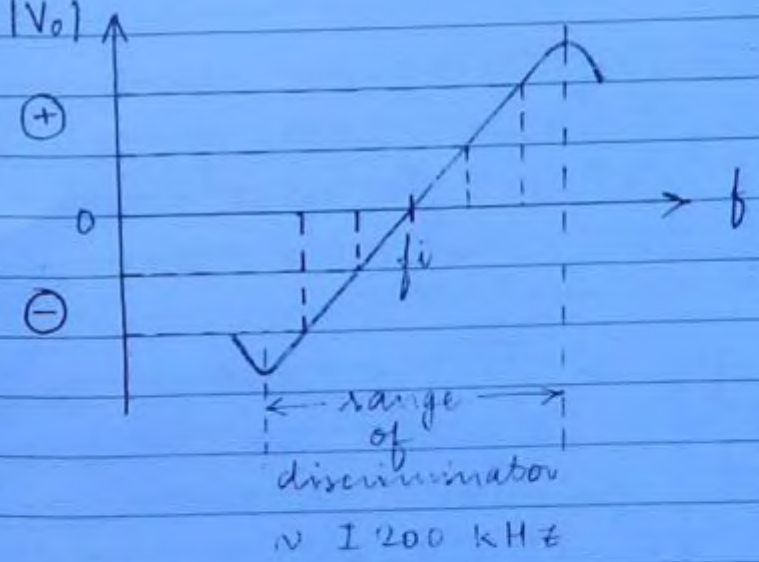
$|V_o| = k(|V_{10}| - |V_{20}|)$

$= -ve$

Discriminator characteristics –



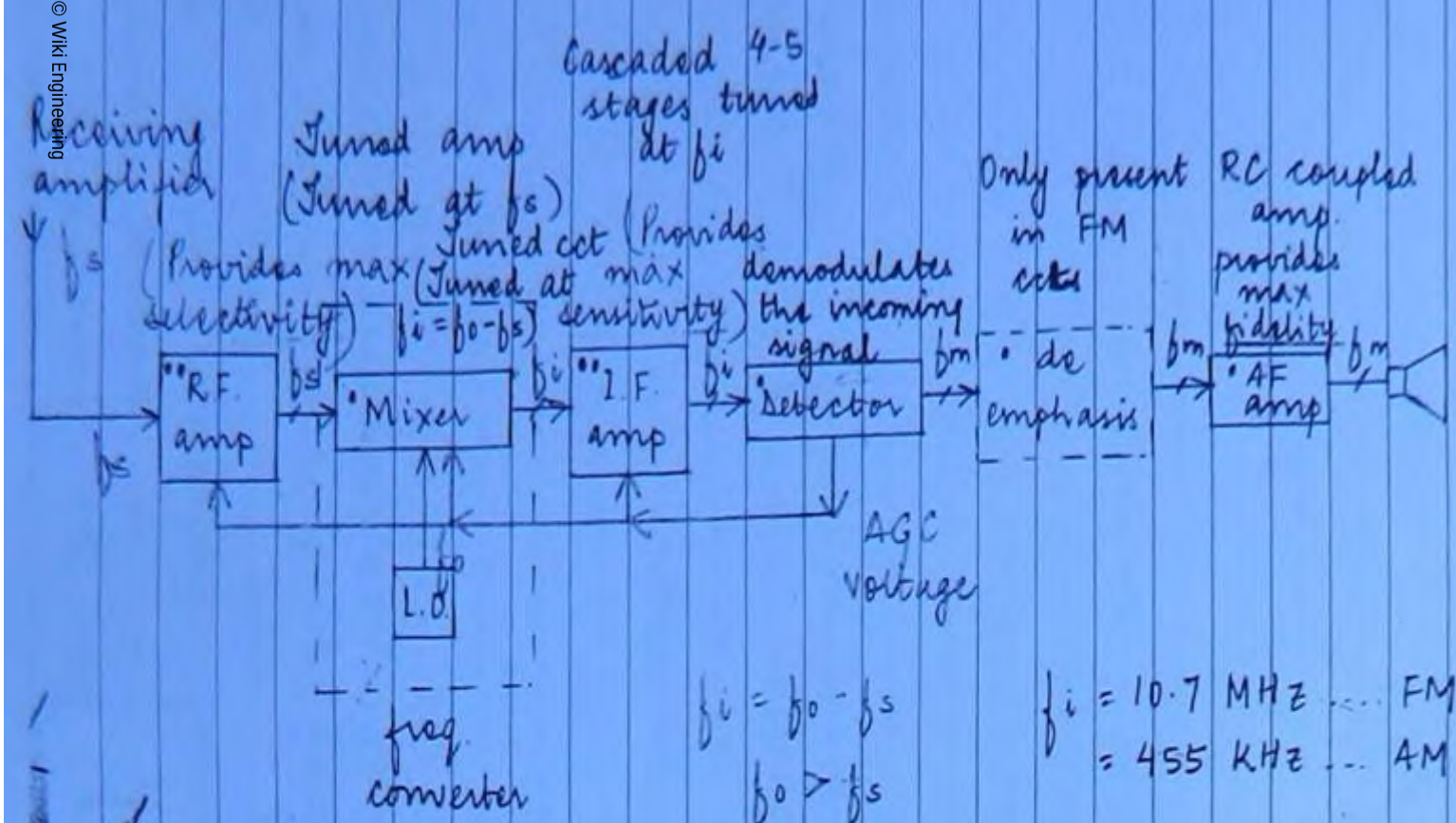




... discriminator characteristics ...(S curve)

In the Foster Seeley Discriminator, the incoming frequencies are being discriminated & the output voltage is obtained which is proportional to the variation in incoming frequencies w.r.t the centre frequency fi. ∴ the original modulating signal is recovered from the incoming FM signal.

Super - hetrodyne radio receiver



$f_i = f_o - f_s$

$f_o > f_s$

$f_i = 10.7$ MHz ... FM

$= 455$ KHz ... AM

anged tuning

(58)

Using this discriminator or WBFM signal can be demodulated depending on the frequency deviation at the input of the network

Why $f_o > f_s$ ... AM

f_s : 535 kHz - 1605 kHz

f_i : 455 kHz

$f_{o(min)} = f_{s(min)} + f_i = 535 + 455 = 990$ kHz

$f_{o(max)} = f_{s(max)} + f_i = 1605 + 455 = 2060$ kHz

$$f_{o(min)} = \frac{1}{2\pi\sqrt{L_o C_{o1}}}$$

$$f_{o(max)} = \frac{1}{2\pi\sqrt{L_o C_{o2}}}$$

$$\frac{f_{o(max)}}{f_{o(min)}} = \sqrt{\frac{C_{o1}}{C_{o2}}}$$

$$\frac{C_{o1}}{C_{o2}} = \left(\frac{f_{o(max)}}{f_{o(min)}}\right)^2 = \left(\frac{2060}{990}\right)^2 \cong 4.4$$

The local oscillator frequency is always kept higher ~~than~~ than signal frequency so that a practical range of variation of the capacitor of the local oscillator is obtained.

The capacitor of RF amplifier, mixer & the local oscillator are ganged together to acheive simultaneously following objectives

i) The receiver is tuned at the desired input signal frequency

ii) f_o is always maintained above the signal frequency f_s

iii) The difference between the local oscillator frequency & the signal frequency must always be maintained equal to the intermediate frequencies

automatic gain

The AGC voltage is used from the detector stage to all the previous stages using negative feedback so that the output of the receiver is constant irrespective of variation in the input signal strength. (52)

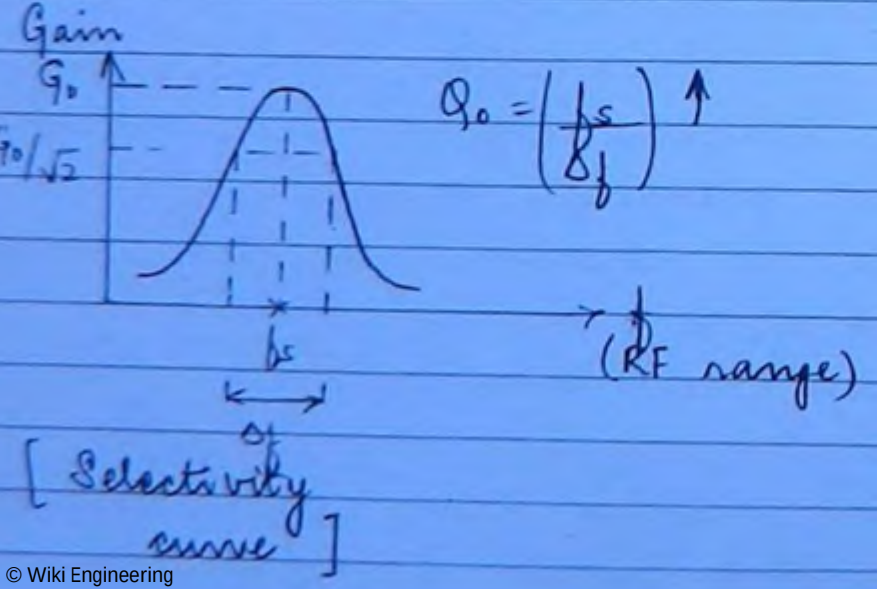
* Functions of RF amplifier -

i) Higher gain & selectivity
ii) Higher SNR
iii) Provides better coupling between the receiving antenna & the rest of the receiver.
iv) Provides higher image signal rejection.

Selectivity -

* This is the ability of the radio receiver to accept desired signal thereby rejecting all unwanted signals.

* Maximum selectivity is provided by the RF amp.



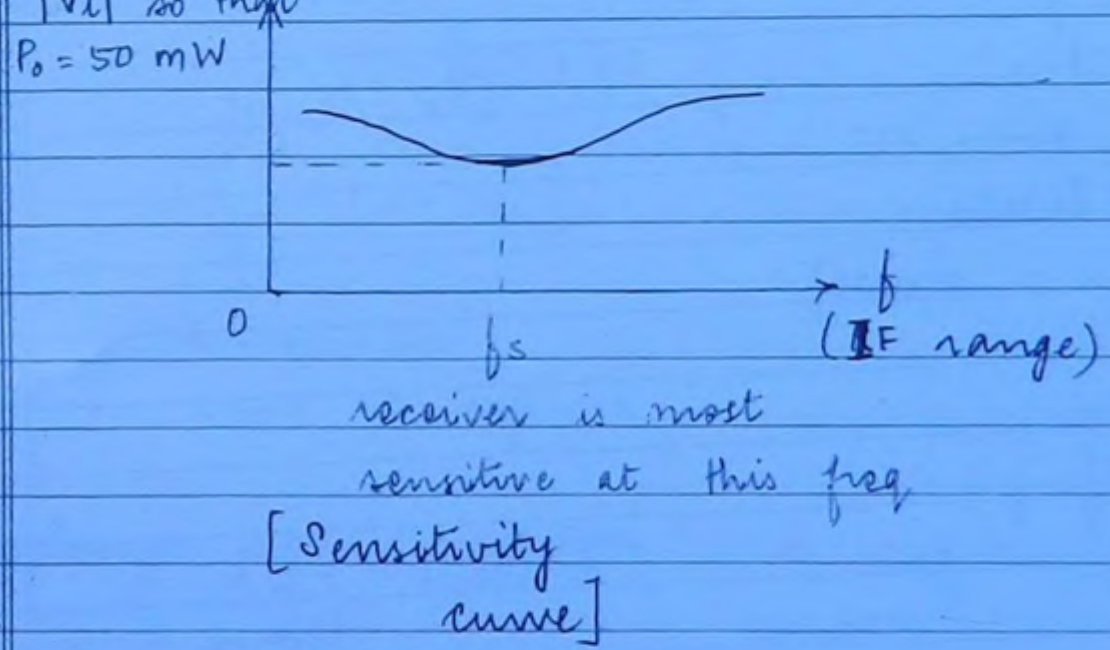
Sensitivity –

* This is the ability of radio receiver to amplify weak signal.

* Sensitivity is defined in terms of the voltage which must be applied to the input of the radio receiver to give standard output of 50 mW

* Maximum sensitivity is produced by IF amp.

* Any good radio receiver should have minimum sensitivity.



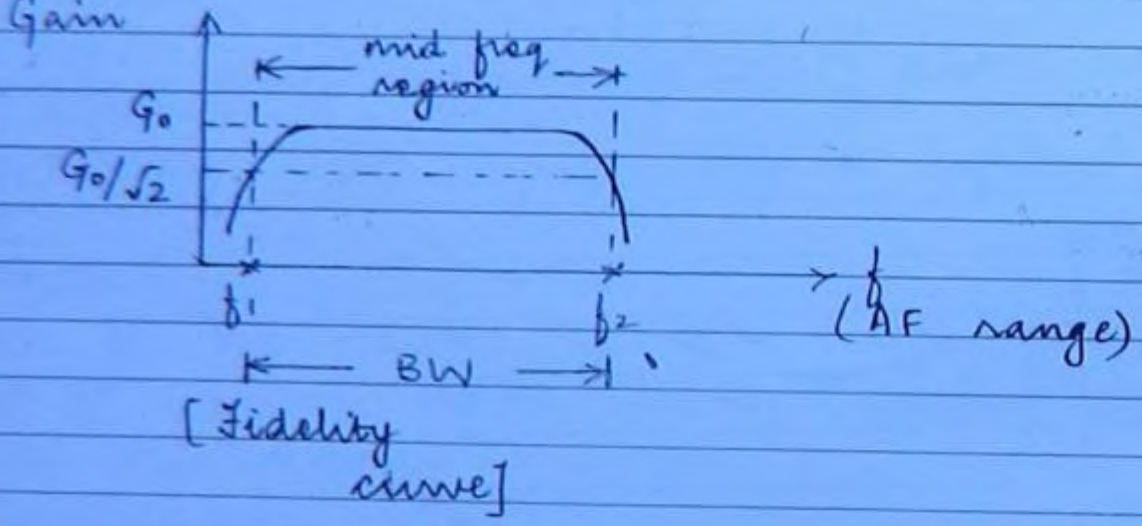
[Sensitivity curve]

Fidelity -

* This is the ability of any radio receiver to reproduce a signal having same quality with which it was transmitted.

* Therefore the frequency response of any amplifier must be flat over entire audio range.

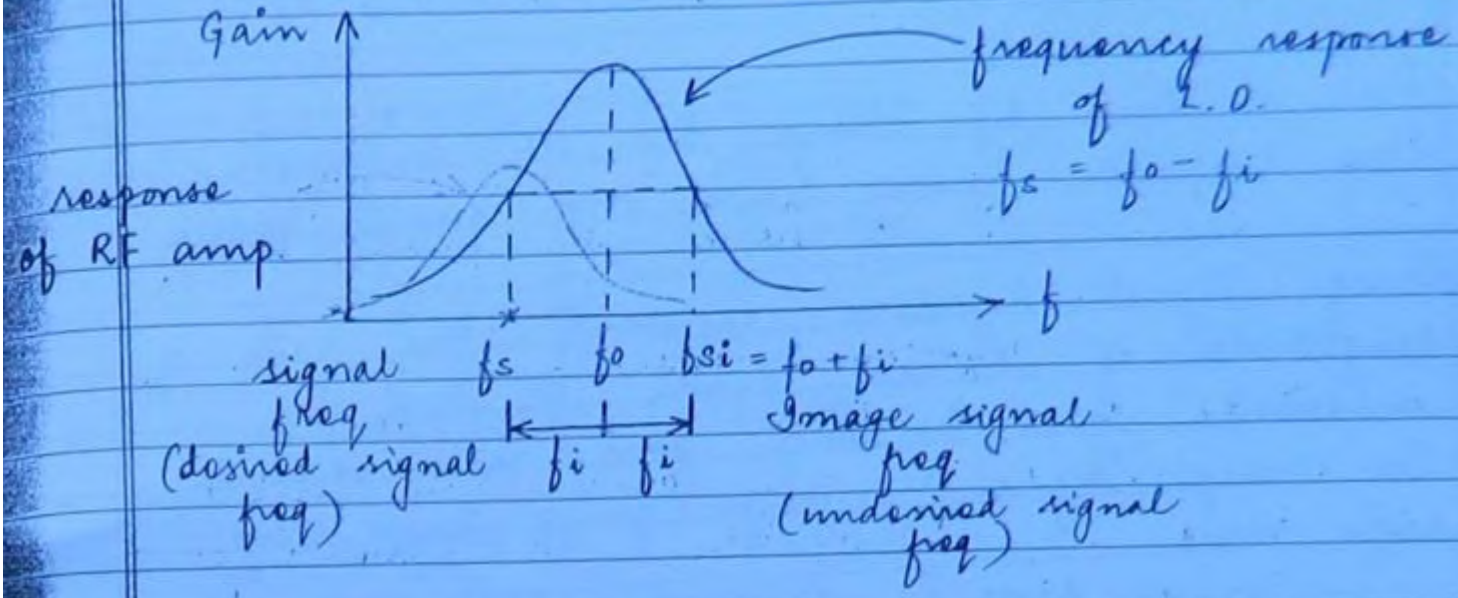
* Maximum fidelity is provided by the AF amp.



[Fidelity curve]

$G_0 \times BW$
$= G_0 \times (f_2 - f_1)$
$\simeq G_0 \times f_2$ ... gain BW product = constant = F.O.M (figure of merit)

Imp Image Signal Frequency -



$$f_{si} = f_o + f_i$$

$$= (f_s + f_i) + f_i$$

$$\boxed{f_{si} = f_s + 2f_i}$$

↑ undesirable ... must be rejected

Image signal rejection

$$\alpha = \sqrt{1 + Q_0^2 \rho^2}$$

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$\alpha_{dB} = 20 \log_{10} \alpha$$

The image signal frequency is undesirable and must be rejected from entering into radio receiver.

This is done by using the RF amplifier. This amplifier must have high quality factor.

Image Signal Rejection represents the ratio of amplitude of desired signal to that of undesired signal. For any good radio receiver, the image signal rejection must have very high value.

Ex: In a broadcast, AM super heterodyne radio receiver, the quality factor of radio receiver is 100. Calculate the image frequency & its rejection at
i) 1000 kHz
ii) 25 MHz

Sol: i) $f_s = 1000$ kHz
$f_i = 455$ kHz

$$f_{si} = f_s + 2f_i = 1000 + 910$$
$$= 1910 \text{ kHz}$$

ii) for 25 MHz $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{ci}} = 1.386$

$$\alpha = \sqrt{1 + Q_0^2 \rho^2} \cong 138.6$$

$$\Rightarrow 20 \log_{10} \alpha = 20 \log_{10} 138.6 \cong 42 \text{ dB}$$

Image signal rejection is Excellent.

iii) $f_s = 25 \text{ MHz}$
$f_i = 455 \text{ kHz} = 0.455 \text{ MHz}$

$f_{si} = f_s + 2f_i = 25.91 \text{ MHz}$

$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = 0.0715$

$\alpha = \sqrt{1 + Q_0^2 \rho^2} \cong 7.22$

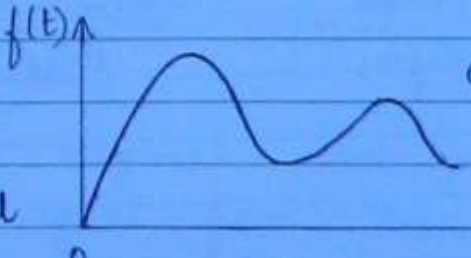
$\Rightarrow 20 \log_{10} 7.22 \cong 17.17 \text{ dB}$

Image ~~Rejection~~ Signal Rejection is poor.

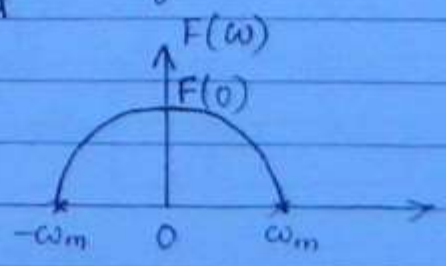
PM (Pulse analog modulation)

- PAM — Pulse amplitude modulation
- PWM — Pulse width modulation
- PPM — Pulse position modulation

Modulating signal → $f(t)$
non-sinusoidal signal

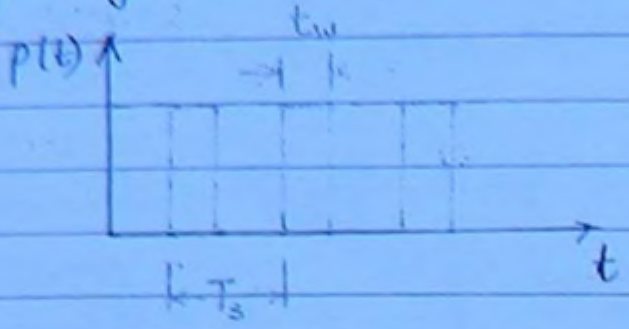


... low pass signal
... band limited signal



ω_m .. max modulating freq comp

Carrier (Sampling signal) → Pulse train.

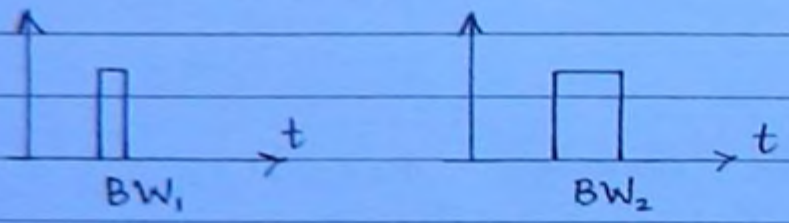


T_s ... sampling interval

$f_s = \frac{1}{T_s}$ .... sampling freq

$\omega_s = \frac{2\pi}{T_s}$

t_w ... pulse width



$\omega \propto \frac{1}{t}$

$BW_1 > BW_2$

$\omega_s \geq 2\omega_m$ Sampling theorem

$f_s \geq 2 f_m$

TDM (Time Division Multiplexing)

In pulse modulation, one of the 3 properties of the sampling signal namely amplitude (or) width (or) position is varied at a time w.r.t the instantaneous value of the amplitude of modulating signal.

The sampling frequency is almost comparable to highest modulating frequency comp. contained in the signal ∴ pulse modulated signal cannot be transmitted through antenna. ∴ long range transmission is not possible. ∴ such signals are then transmitted over a transmission line. Hence a range of transmission is limited.

Using PM, TDM is possible ∴ large number of signals can be transmitted in the time domain simultaneously over a common communication channel. using single sampling signal.

Sampling Theorem -

... for low pass or band limited signals

For the recovery of original low pass or band limited modulating signal from the sampled version the sampling frequency of the sampling signal must be generated greater than or equal to twice of highest modulating freq comp. contained in the signal

$$\omega_s \geq 2\omega_m$$

$$f_s \geq 2f_m$$

$$f_{s(min)} = f_{s(Nyquist)} = 2f_m$$

$$T_s = \frac{1}{f_{s(Nyquist)}}$$

The nyquist rate of sampling represents the minimum rate at which modulating signal is being sampled. If the sampling is done at a rate lower than the nyquist rate the original signal can never be recovered.

-13.

Ex $f(t) = 20 \cos 20\pi t \cdot \cos 200\pi t$

To find

$f_{s(min)}$

T_s

Sol $f(t) = 10[\cos \underbrace{220\pi t}_{\omega_{m_1}} + \cos \underbrace{180\pi t}_{\omega_{m_2}}$

$$f_{m_1} = \frac{220\pi}{2\pi} \qquad f_{m_2} = \frac{180\pi}{2\pi}$$

$$= 110\ Hz \qquad = 90\ Hz$$

$$= f_{m(max)}$$

$$f_{s(min)} = f_{s(nyquist)} = 2f_{m(max)}$$

$$= 2 \times 110$$

$$= 220\ Hz.$$

$$T_s = \frac{1}{f_{s(min)}} = \frac{1}{220}\ sec.$$

$f(t) = 10 \sin 20\pi t \cdot \cos 400\pi t + 4\,Sa^2 2000\pi t$

To find

$f_{s(min)}$

T_s

$$f_m = \frac{1}{2\pi}(400\pi \pm 20\pi)$$

$$= \begin{cases} 210\ Hz \\ 190\ Hz \end{cases}$$

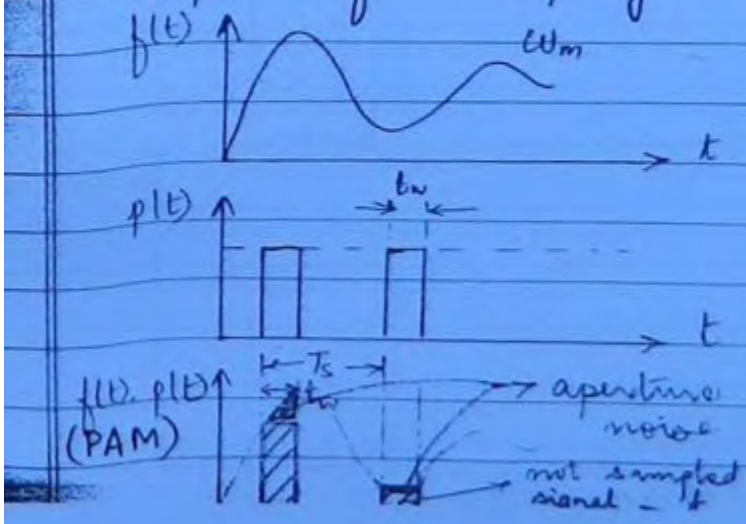
$$Sa^2(x) = \left(\frac{\sin x}{x}\right)^2$$

$$f_m = \frac{4000\pi}{2\pi} = 2\ kHz \quad \text{--- } f_{m(max)}$$

$$f_{s(min)} = 2 f_{m(max)}$$
$$= 2 \times 2$$
$$= 4\ kHz$$

$$T_s = \frac{1}{f_{s(min)}} = \frac{1}{4K} = 0.25\ msec$$

Principle of sampling -



... modulating signal

$f_s \geq 2 f_m$

$\omega_s \geq 2\omega_m$

... Pulse train

... Sampling signal

$T_s = \frac{1}{f_s}$

... Sampled signal

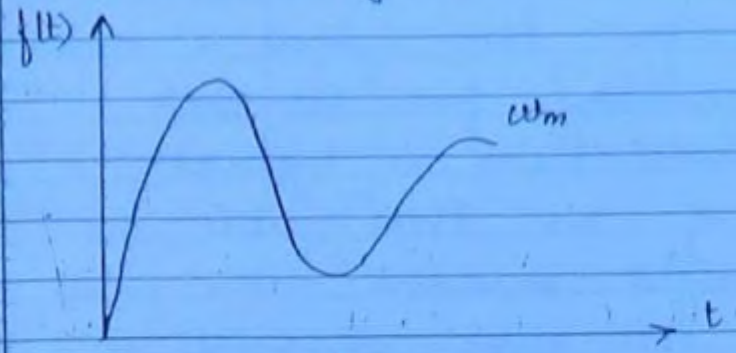
- The sampled signal maybe obtained using flat topped sampling which is practically simpler to generate.

- During flat topped sampling the aperture noise will always exist.

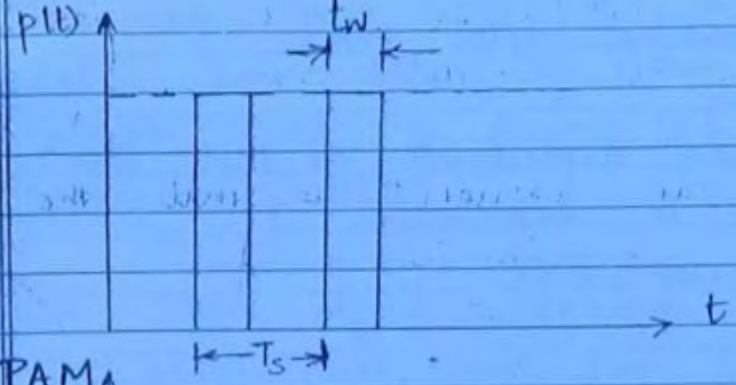
- Using sampling we can then transmit large number of signals simultaneously using a common sampling frequency over a common communication channel. This process represents time division multiplexing which has much simpler circuitry as compared to the FDM system.

- During pulse modulation one of the 3 properties of the pulses namely
  i) amplitude
  ii) width
  iii) position
  is varied at a time with the instantaneous value of amplitude of modulating signal resulting in PAM, PWM & PPM signals.

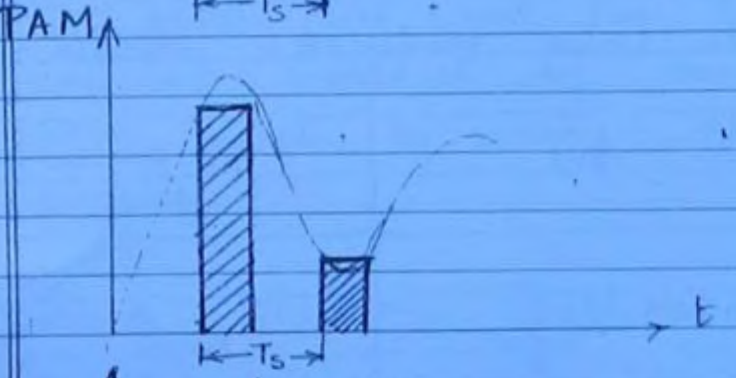
Graphical representation of various pulse modulated signals -



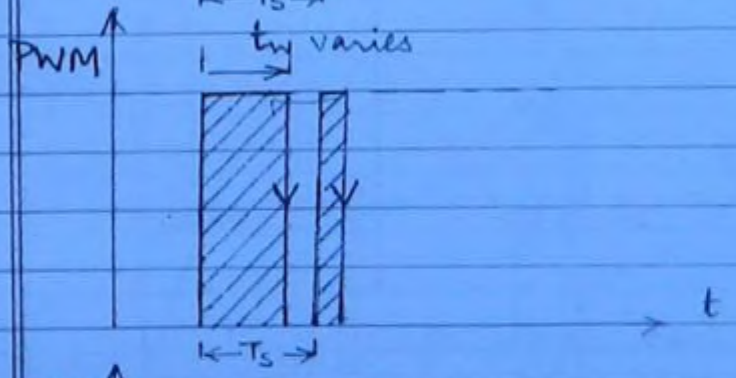
... modulating signal
... max mod freq = ω_m ; f_m



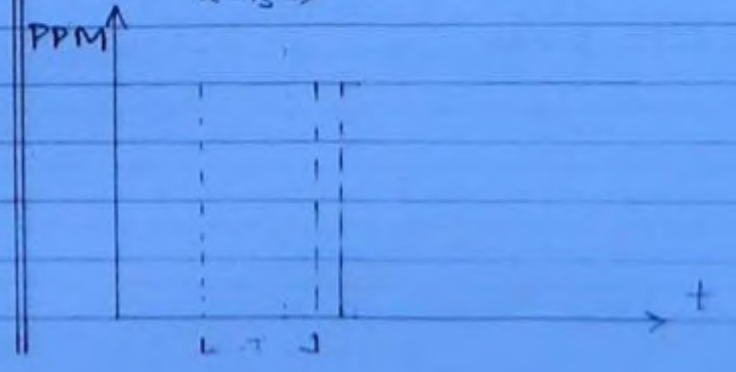
... sampling signal
$f_s \geq 2f_m$; $T_s = 1/f_s$.



... Pulse amplitude modulated signal



... Pulse width modulated signal



... Pulse position modulated signal

* The circuit configuration of the PAM signal is the simplest using a Sample & Hold circuit.

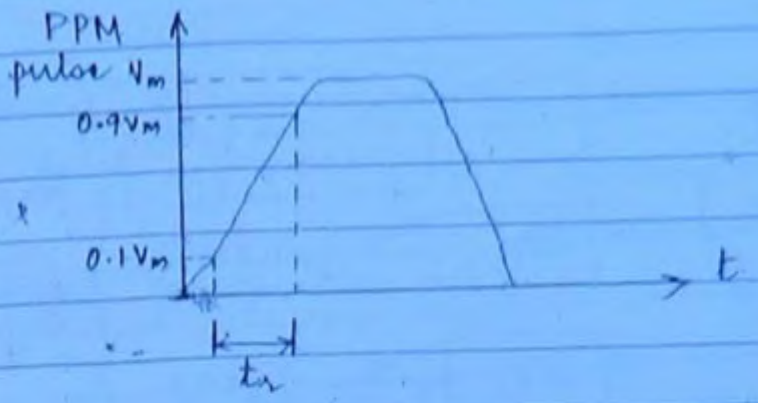
* The circuitry required for PPM system is most complex & can be generated from the PWM signal A monostable multivibrator is triggered at the falling edges of the PWM pulses the result is constant height, constant width pulses where shift of each PPM pulse corresponds to the width of corresponding PWM pulse.

* PAM signal is amplitude dependent so that the contribution of noise is maximum & therefore it has lowest SNR.

Comparison of various Pulse modulated signals -

	circuit complexity	BW	SNR
PAM	min	$\sim 2f_m$	min
PWM		$\sim 5f_m$	
PPM	max	$\sim 1/t_r$	max

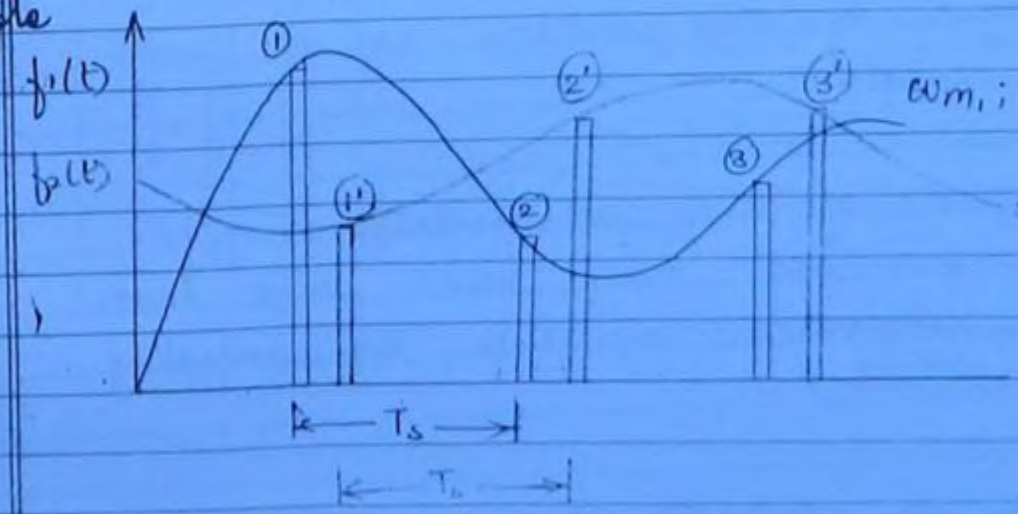
t_r = rise time of PPM pulse



The rise time (t_r) of a PPM pulse represents total time taken by the pulse to increase its voltage level from 10% to 90% of its maximum value.

The PAM signal requires least complex circuitry & lowest BW but has low SNR. The SNR can be increased by converting PAM pulses to PCM signal by using quantisation & encoding processes.

Time Division Multiplexing -

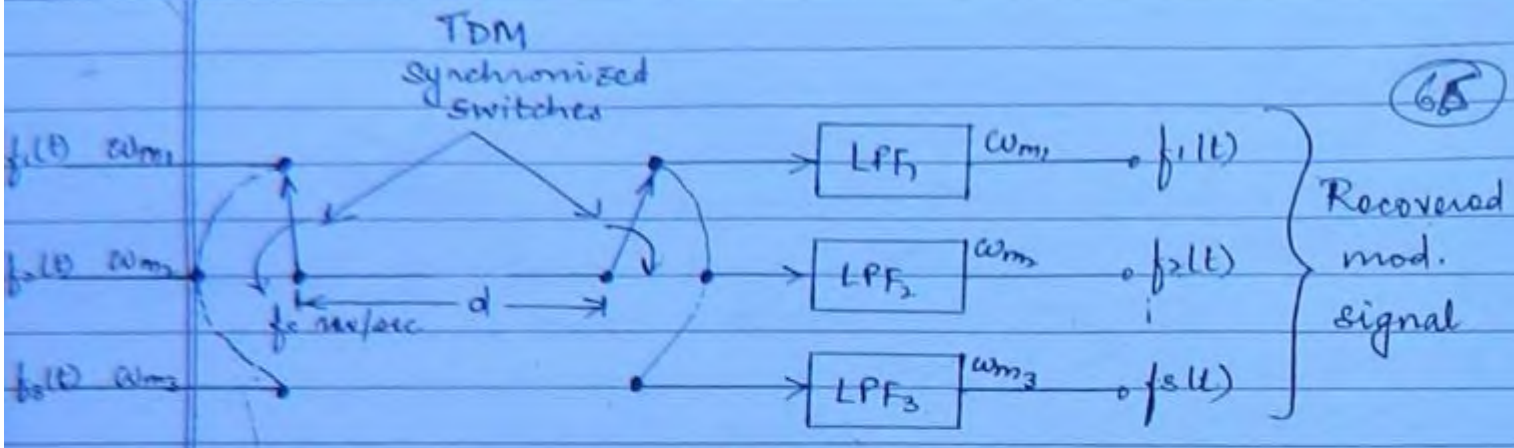


ω_{m_1}, f_{m_1} $T_s = 1/f_s$

ω_m

$\omega_{m_1} \geq \omega_{m_2}$

$\omega_s \geq 2\omega_m$



commutator (MUX)

de commutator (DE-MUX)

$$t_d = \frac{d}{v_p}$$ .... time delay between 2 synchronized switches

* The switch at the commutator & at the decommutator must be synchronised & must have a time delay of t_d which depends upon the length of the channel & velocity of propagation of pulses through the channel.

The contact time of the switch controls width of each sample.

Since the speed of the commutator is fs in revolutions per sec then total time taken to make one revolution represents the sampling interval t_s.

Each LPF has a cut-off frequency same as the highest modulating frequency component of the corresponding signal.

The multiplexed signals are transmitted over a common communication channel with a common sampling frequency.

(63)

Design parameters -

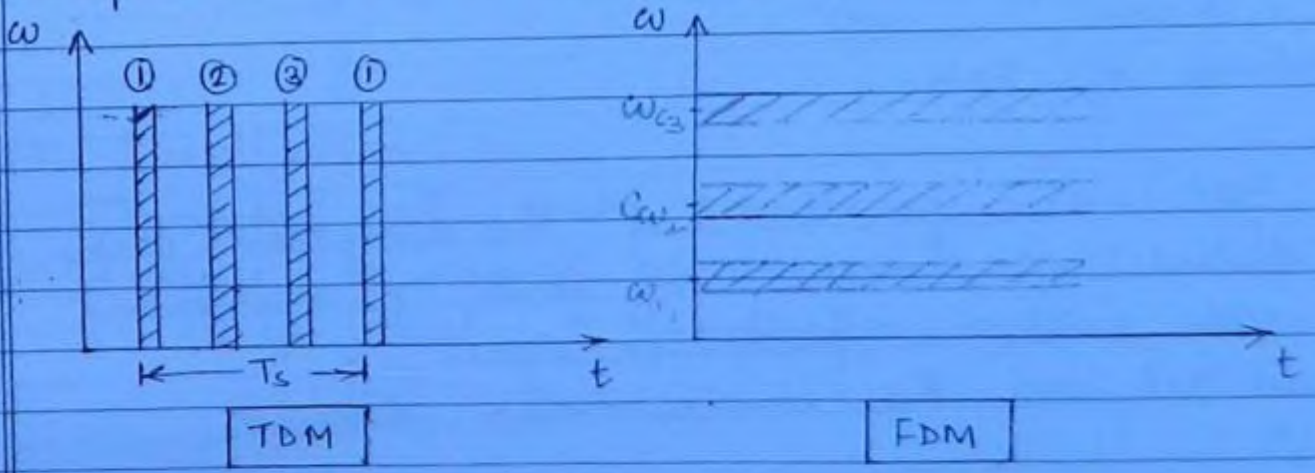
$t_w \downarrow \rightarrow m \uparrow$
$\rightarrow BW \uparrow$

m = no. of signals being multiplexed on TDM basis.

$T_s \uparrow \rightarrow m \uparrow$
$\rightarrow f_s = \frac{1}{T_s} \downarrow \Rightarrow f_s \geq 2f_m$

* The width of each sample & sampling interval are adjusted depending upon
i) no. of signals to be multiplexed.
ii) availability of BW for the given system

Comparison between TDM & FDM -



Some features of TDM system -

1. In TDM, the entire time interval is divided into smaller time slots & corresponding to each time slots a sample from one particular system is transmitted over a common comm. channel.

2. A common sampling freq is used.

3. Relatively low SNR.

4. The range of transmission is limited since the signal is transmitted over a Tx line.

5. A synchronising pulse is required in each revolution of the multiplexer & demultiplexer of the TDM system.

6. Used in pulse modulation or digital modulation like PCM system.

Features of FDM -

1. In FDM, entire freq band of the broadcast range is divided into smaller freq band & corresponding to each band a particular signal is transmitted over the communication channel.

2. A separate carrier freq & ∴ a separate transmitter is required for each signal.

3. Long range transmission is possible since the transmission is through antenna.

4. More complex circuitry

5. The SNR may be increased by increasing the transmitted power.

6. Used for analog communication system like AM & FM

DIGITAL COMM SYSTEMS

(Pulse Code Modulation) (PCM)

Principle -

modulating signal

⇓

1. sampling process $f_s \geq 2f_m$, $T_s = 1/f_s$ } Sample & Hold circuit

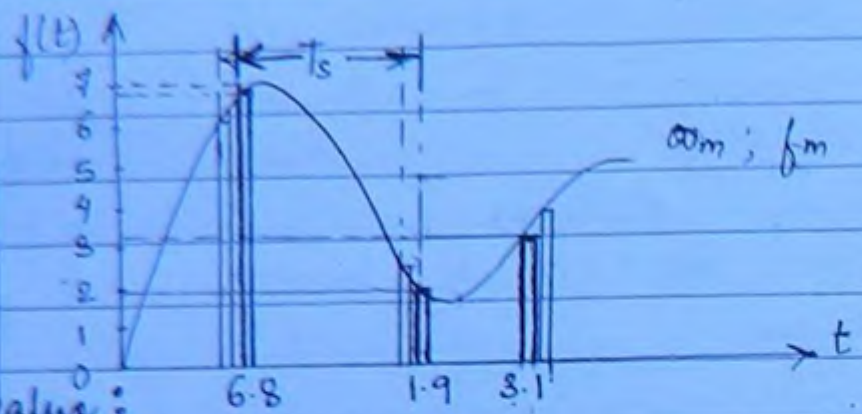
⇓

2. Quantization :
Rounding off each sample value to nearest pre-determined voltage level ⇒ Quantisation noise } A/D converter (SAR type A/D)

⇓

3. Encoding ⇒ Binary coding
$Q = 2^n$ ... Quantization level

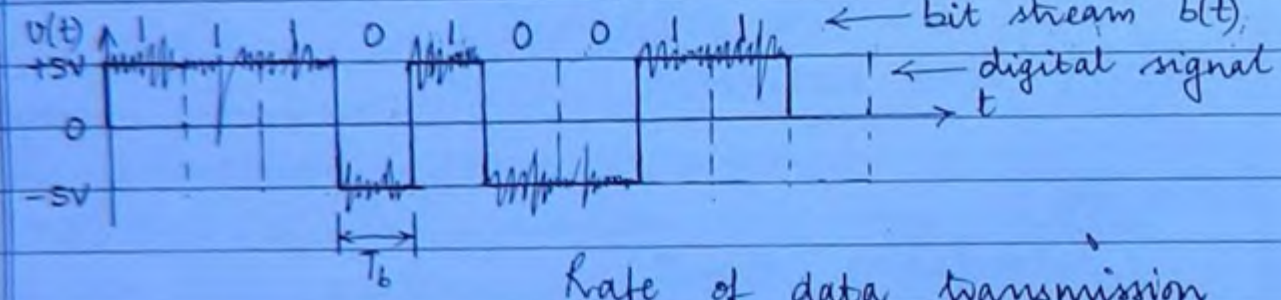
Generation of PCM signals -



Sample value : 6.8 1.9 3.1

Quantized value : 7 2 3

Coded value : 111 010 011



Rate of data transmission

$$r_b = \frac{1}{T_b} \quad \text{bps}$$

Features of digital signals -

1. The detection of the given signal does not depend upon the absolute value of the voltage received. But it depends only upon the ~~relative~~ related voltage received w.r.t the threshold value.

2. Very low probability of error. Error in the detection takes place only when the noise exceeds the threshold value, at the same time instant at which detection is done.

3. 2 level signal ∴ noise can always be minimised using a limiter circuit. ∴ such signal has very high SNR.

Transmission of PCM signal -

1. Since the signal has been sampled at the sampling frequency such signal cannot be transmitted through antenna. ∴ such signal is transmitted through transmission line ∴ the range of transmission is limited.

2. The rate of data transmission depends upon
   i) SNR. on the channel.
   ii) Availability of BW.

   ≈ 64 kbps ... Cu cond. wire
   ≈ 1 G bps } ... optical fibre
   0.5 G bps }

System Performance -

n = no. of bits per sample
= bit coding parameter.

Q = no of quantization levels.

$$\boxed{Q = 2^n}$$

V_m = Peak value of signal
$V_{pp} = 2V_m$
S = Step size

$$\boxed{S = \frac{V_{pp}}{Q} = \frac{2V_m}{2^n}}$$

N_q = Quantization noise (power)

$$N_q = \frac{S^2}{12}$$

$\left(\frac{S_o}{N_q}\right)$ ... signal to quantization noise ratio.

$$\frac{S_o}{N_q} = \frac{3}{2} Q^2 = \frac{3}{2} \cdot 2^{2n}$$

$$\left(\frac{S_o}{N_q}\right)_{dB} = 10 \log_{10} \left(\frac{3}{2} \cdot 2^{2n}\right)$$

$$\cong 1.76 + 6n \quad \text{.... dB}$$

f_s = sampling freq
f_m = max. mod freq

$$f_s \geq 2 f_m$$

$$f_{s(min)} = 2 f_m$$

r_b ... bit rate.

$$r_b = n f_s \quad \text{... bps}$$

n ... no. of bits / sample
f_s ... samples / sec

BW reqd $$BW = \frac{1}{2} r_b = \frac{1}{2} n f_s \quad \text{... Hz}$$

$$BW_{min} = n f_m$$

An audio signal is given by

$$s(t) = 3\cos(1000\pi t) + 3\cos(1400\pi t) + 2\cos(2000\pi$$

The audio signal is quantized using 10 bit PCM signal. Find (73)

i) nyquist rate of sampling
ii) step size
iii) signal power
iv) quantization noise power
v) SNR

$$s(t) = 3\cos(1000\pi t) + 3\cos(1400\pi t) + 2\cos(2000\pi t)$$

$$f_{m(max)} = \frac{2000\pi}{2\pi}$$

$$= 1\text{ kHz}.$$

(i) $f_{s(min)} = 2 f_{m(max)}$

$= 2\text{ kHz}$

$$V_{RMS} = \sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = \sqrt{11}$$

(ii) $S_0 = V_{RMS}^2 R = 11\text{ W}$ $(R = 1\text{ ohm})$

(iii) $S = \frac{V_{pp}}{Q} = \frac{2(\sqrt{2}\, V_{RMS})}{2^n} = \frac{2(\sqrt{2}\cdot\sqrt{11})}{2^{10}}$ V

$=$

(iv) $N_q = \frac{S^2}{12}$ W $=$

(v) $\frac{S_0}{S_q} \quad \frac{S_0}{N_q} = \frac{3}{2}\cdot 2^n = \frac{3}{2}\cdot 2^{20}$

Ex Consider a binary PCM transmission of a video signal with sampling frequency of 10MHz or 10M samples/sec. Calculate the bit rate required to obtain a minimum SNR of 45 dB.

Sol. $f_s = 10\,MHz$
$= 10\,M$ samples/sec.

$\left(\frac{S_o}{N_q}\right)_{min} = 45\,dB$

$\left(\frac{S_o}{N_q}\right) \geq 45\,dB$

$R_b = nf_s$
$= n \times 10M$

$1.76 + 6n \geq 45$
$n \geq \frac{43.24}{6}$
$n \geq 7.2$
Select $n = 8$

$R_b = 8 \times 10$
$= 80\,Mbps.$

The BW of TV + audio signal is 4.5 MHz. The signal is converted to PCM signal with 1024 quantisation levels. Find required bit rate assuming signal is sampled at a rate 90% above nyquist rate.

(75)

$f_m = 4.5\,MHz$

$f_{s(min)} = 2f_m = 9\,MHz = 9\,M$ sample/sec

$Q = 1024 = 2^{10} = 2^n ; \quad n = 10.$

$r_b = nf_s.$

$= 10. \times \left(\frac{90}{100} + 1\right) f_{s(min)}$

$= 108$ M bps.

Find the nyquist BW & SNR of a PCM system sampling at 8 kilo samples per sec. & using 6 bits per words for transmission. What will be improvement in the system performance if the channel BW is increased to a factor of 4/3.

case 1 -

$n = 6$

$f_s = 8k$ samples/sec.

$BW = \frac{1}{2} r_b = \frac{1}{2} nf_s = 24$ kHz.

$\left(\frac{S_0}{N_q}\right)_{dB} \simeq 1.76 + 6n$

$\simeq 1.76 + 36$

$\simeq 37.76$ dB

Case 2.

$$BW' = \frac{4}{3} BW$$

(78)

$$= \frac{4}{3} \times 24$$

$$= 30 \text{ kHz} = \frac{1}{2} n' f_s \qquad n' = \frac{32}{8} \times 2 = 8$$

$$\left(\frac{S_o}{N_q}\right)_{dB}' \simeq 1.76 + 6n' =$$

$$\simeq 1.76 + 48$$

$$\simeq 49.76 \text{ dB}.$$

Improvement in SNR = 49.76 - 37.76

= 12 dB.

Digital carrier modulation -

modulating signal } ... digital signal (output of PCM system) ⇒ 1010

carrier ... sinusoidal

high freq carrier

$A \sin \omega t$

$A \cos \omega t$

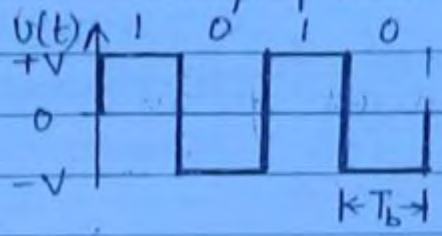
digital carrier modulated signal

- ASK (OOF)
- (B) FSK
- (B) PSK

Any digital signal cannot be transmitted through antenna since the sampling frequency has a relatively low value. For long range transmission of the signal through antenna. A high frequency carrier is modulated using the digital signal resulting in digital carrier modulated signal.

Digital carrier modulated signals -

... Graphical representation.

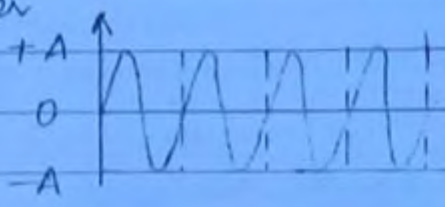


... modulating signal

... output of PCM system.

Bit rate $r_b = 1/T_b$

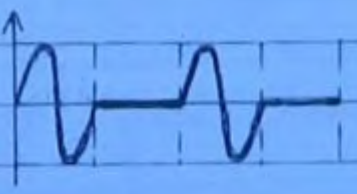
carrier



... high freq sinusoidal carrier

$A \sin \omega_c t$

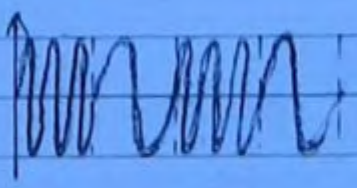
ASK



... Amplitude shift keying (ASK) signal

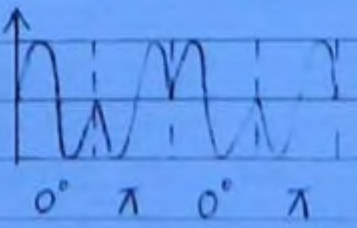
... On-off keying (OOK) signal

BFSK



... (Binary) freq shift keying signal

BPSK



... (Binary) phase shift keying signal

ASK -

1. Simple circuitry required for its generation.
2. Minimum signal to noise ratio since the signal is amplitude dependent.
3. Relatively high probability of error in the detection of the signal.
4. Relatively smaller BW required for its transmission
5. Used for telegraphy & teleprinting of transmission of data.

BPSK -

1. Relatively complex circuitry required for its generation.
2. Relatively high SNR
3. Relatively low probability of error
4. Relatively higher BW required for the transmission of the signal
5. Used for satellite communication in its modified form as QPSK (Quadrature phase shift keying) for which the circuitry required is much more complex but BPSK system requires half BW as compared to PSK system

BFSK -

1. More complex circuitry required for the transmission of the signal
2. High SNR
3. Very low probability of error
4. Large BW
5. Used for mobile communication in its modified form as GMSK (Gaussian Minimum Shift Keying)

DELTA MODULATION (DM)

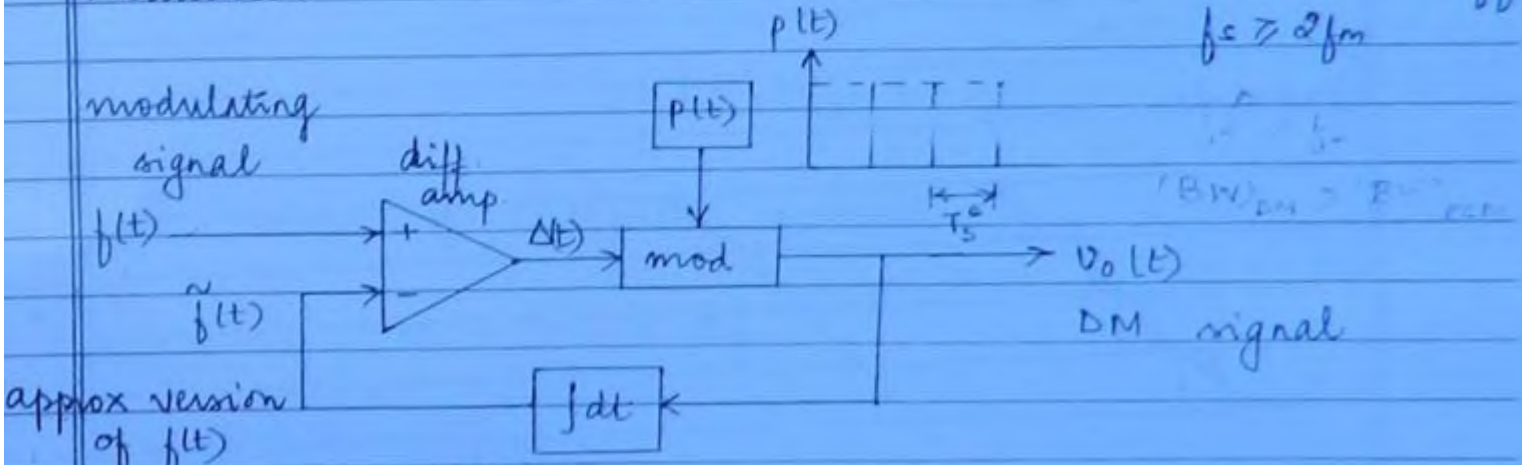
... digital modulation system

The PCM system becomes more complex when the number of quantization levels increase or the bit coding parameter increases. To avoid the use of quantizer & to simplify the circuit configuration, a delta modulation system is used. Such system has much less complex circuitry but the requirement of BW is more for DM system as compared to PCM system. Since both systems are digital systems ~~the~~ both systems have comparable SNR.

$f_s = \frac{1}{T_s}$ N 3 or 4 times of f_s

$f_s \geq 2f_m$

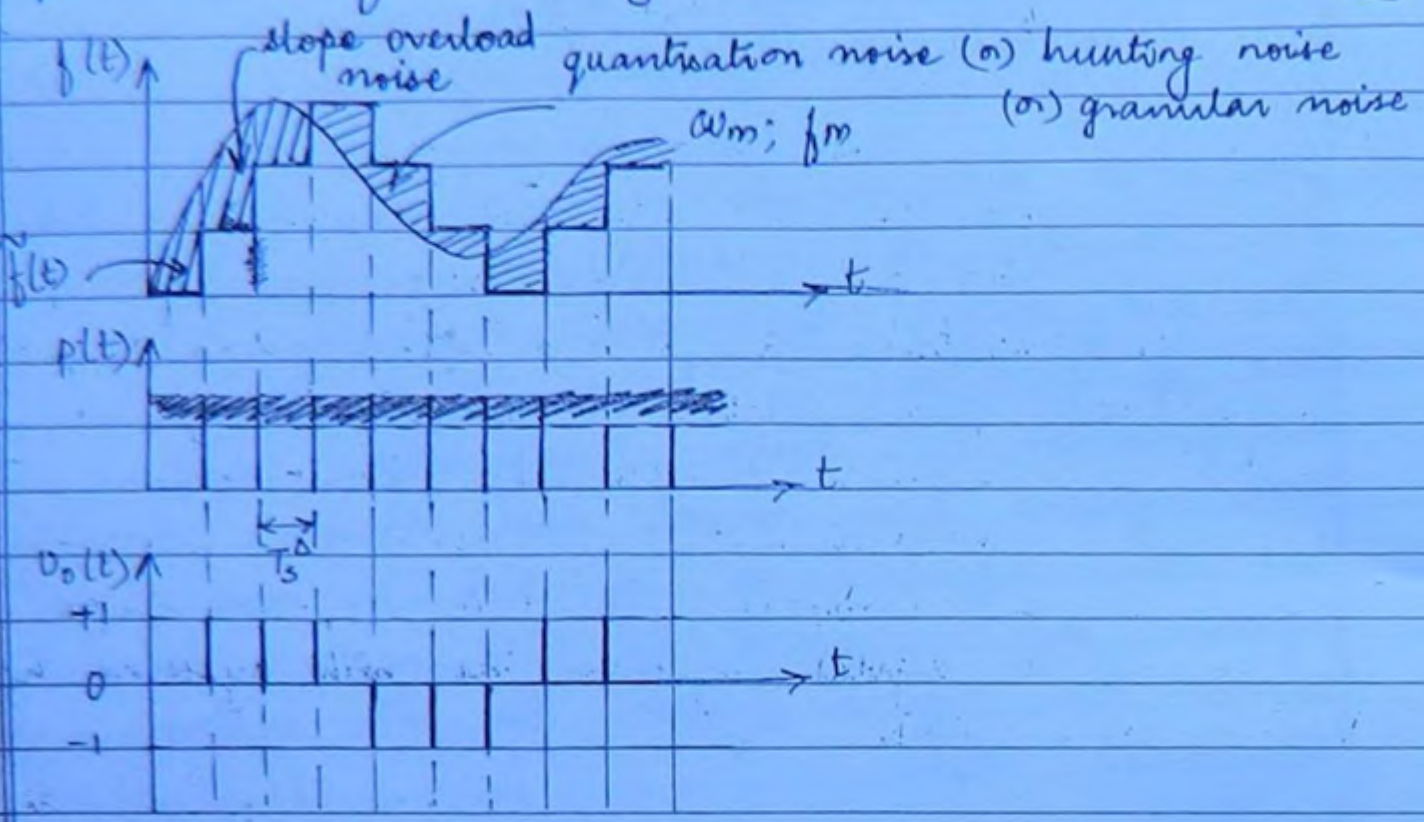
Modulator



algorithm

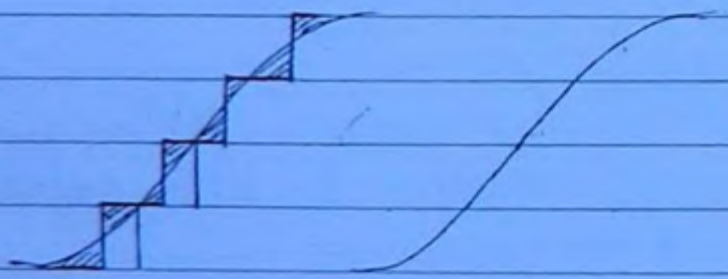
1. $\Delta(t)$ = +ve ; $f(t) > \tilde{f}(t)$; $v_o(t)$ = +1 pulse
2. $\Delta(t)$ = -ve ; $f(t) < \tilde{f}(t)$; $v_o(t)$ = -1 pulse

Generation of DM signal -



To minimise -

1. Granular noise

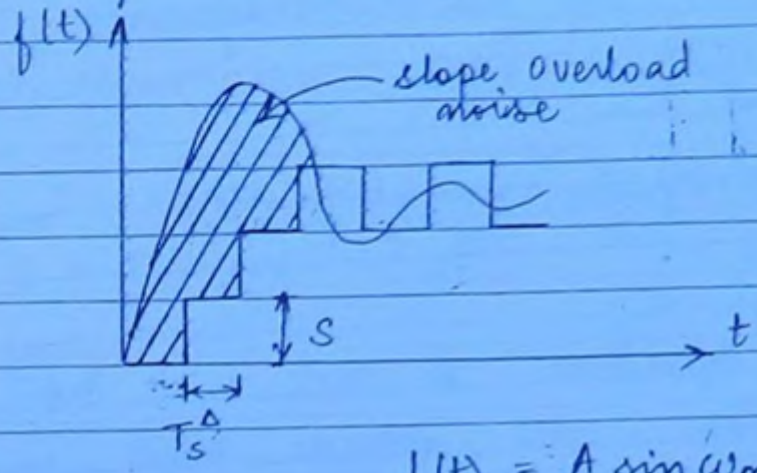


(a) $S \downarrow$ → range of Transmission ↓

(b) $T_s^\Delta \downarrow$ → $f_s^\Delta \uparrow$ N 3 or 4 times of f_s

→ BW ↑

Q. Slope overload noise - condition to avoid slope overload noise -



$$f(t) = A \sin \omega_m t$$

... modulating signal

$$\text{slope } \frac{df(t)}{dt} = \omega_m A \cos \omega_m t \equiv m_1$$

$$|m_1| = 2\pi f_m A$$

slope of integrator output

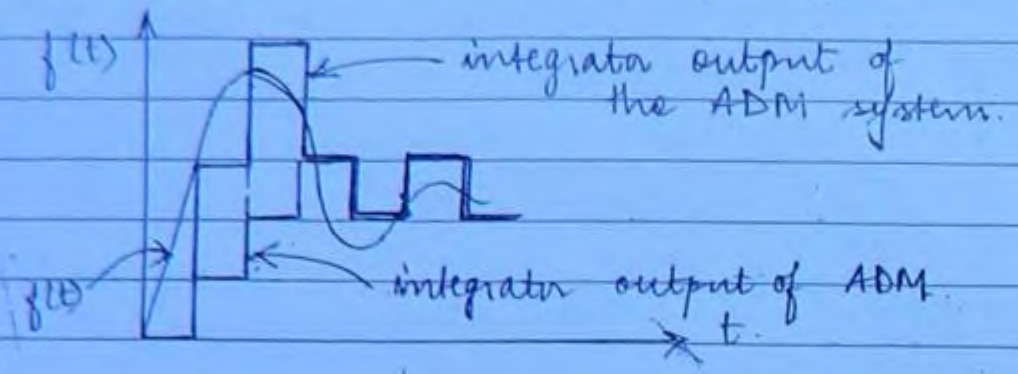
$$m_2 = \frac{S}{T_s^\Delta} = S f_s^\Delta$$

To avoid slope overload noise

$$2\pi f_m A \leq S f_s^\Delta \Longleftarrow$$

ADAPTIVE DELTA MODULATION - (ADM)

... DM with variable step size



* If the specified condition is not met then the slope overload noise will always exist in the DM system.

* To minimise slope overload noise for this condition we use a delta modulator with variable step size or adaptive step size. This is done by including a variable gain amplifier just before the integrator in the feedback loop.

Ex: A signal of 1V amplitude and 800 Hz frequency is transmitted by a DM system. Find minimum step size so that the slope overload is avoided if the sampling frequency is 40 K.samples/sec

Sol.

$A = 1V$

$f_m = 0.8\ kHz$

$f_s^{\Delta} = 40\ K.samples/sec$

$$2\pi f_m A \leq \delta f_s^{\Delta}$$

$$\delta \geq \frac{2\pi f_m A}{f_s^{\Delta}} \Rightarrow \delta_{min} = \frac{2\pi f_m A}{f_s^{\Delta}} = 0.1256\ V$$

Ex: A DM system is designed to operate at 8^3 times the nyquist rate for a signal with 3 kHz BW. The step size is 250 mV. Find the maximum amplitude of 1 kHz sinusoidal signal for which the DM system does not show any slope overload.

Sol:

$f_m' = 3\text{ kHz} = W \Rightarrow f_s = 2f_m' = 2W$

$= 6\text{ kHz}$.

$\delta = 250\text{ mV}$

$f_m = 1\text{ kHz}$

$f_s^{\Delta} = 3f_s$

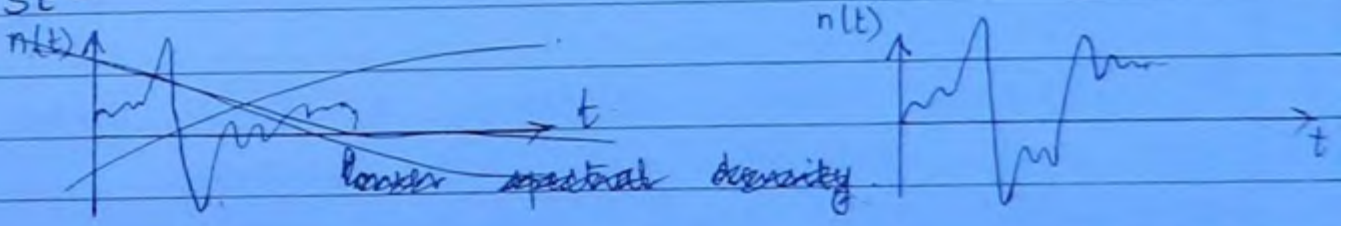
$= 18\text{ kHz}$.

$$2\pi f_m A \leq \delta f_s^{\Delta}$$

$$A \leq \frac{\delta f_s^{\Delta}}{2\pi f_m}$$

$$A_{max} = \frac{\delta f_s^{\Delta}}{2\pi f_m} = 716.2\text{ mV}.$$

NOISE -



Any noise voltage is random in nature & represents an unwanted signal. The noise can then be represented in terms of power spectral density (PSD)

↓

Power per unit BW

... W/Hz

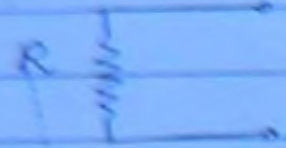
Thermal noise

... Johnson's noise

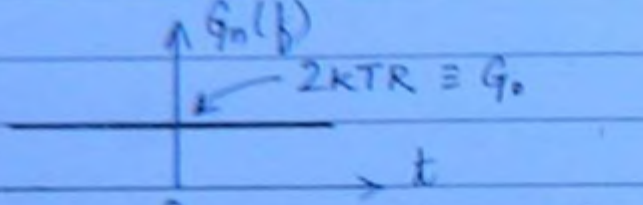
$G_n(f)$ ... PSD of thermal noise

$G_n(f) \propto R$

$\propto T$



$$\boxed{G_n(f) = 2kTR} \quad W/Hz$$



... White noise

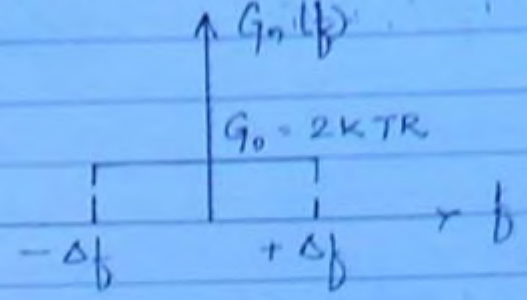
$$N = \int_{-\infty}^{\infty} G_n(f)\, df$$

$$= \infty$$

$$BW = \infty$$

* Any thermal noise represents a white noise since it contains all the frequency components with equal magnitude. Such noise requires infinite amount of power for its generation and infinite amount of BW for its transmission. Any white noise is only mathematical representation which has no practical relevance.

Band limited thermal noise –



Band limited white noise

$$G_n(f) = 2kTR \quad ; \quad -\Delta f < f < +\Delta f$$

$$N = \int_{-\infty}^{\infty} G_n(f)\,df = \int_{-\Delta f}^{+\Delta f} 2kTR\,df = 4kTR\,\Delta f$$

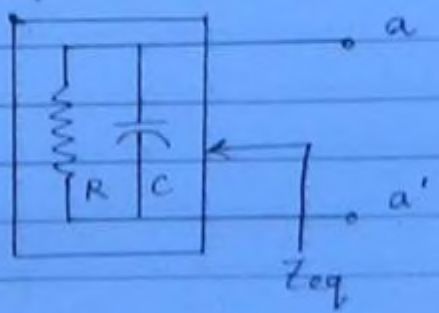
$G_n(f) = 2kTR$	W/Hz	$-\Delta f < f < \Delta f$
$N = 4kTR.\Delta f$	W	
$V_{rms} = \sqrt{4kTR.\Delta f}$	V	
$BW = \Delta f$	Hz	

For the RC LPF shown find the pdf b/w 2 external terminals:

i) PSD

ii) Power contained over entire range of frequencies.

iii) Rms noise voltage

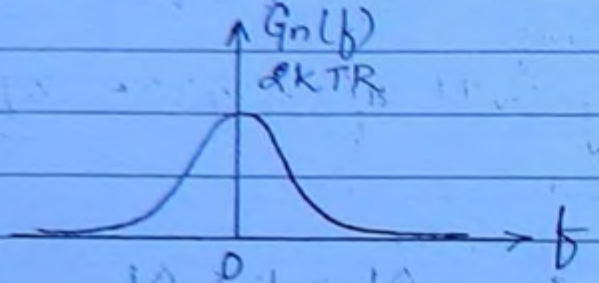


$$Z_{eq} = \frac{R \times 1/j\omega C}{R + 1/j\omega C} = \frac{R}{1 + j\omega RC} \times \frac{1 - j\omega RC}{1 - j\omega RC}$$

$$R_{eq} = Re[Z_{eq}] = \frac{R}{1+(\omega RC)^2}$$

$$G_n(f) = 2kTR_{eq}$$

$$\boxed{G_n(f) = \frac{2kTR}{1+(\omega RC)^2}} \quad \text{.... coloured noise}$$



noise power

$$N = \int_{-\infty}^{\infty} G_n(f)\, df$$

$$= \int_{-\infty}^{\infty} \frac{2kTR}{1+(\omega RC)^2}\, df$$

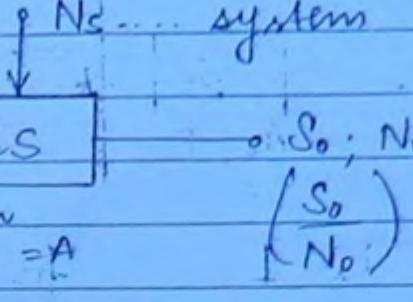
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2kTR}{1+(\omega RC)^2}\, d\omega$$

$\omega RC = \tan\theta$

$RC \cdot d\omega = \sec^2\theta\, d\theta$

$$N = \frac{kT}{C} \qquad V_{rms} = \sqrt{\frac{kT}{C}}$$

Noise figure of a communication system -



$S_o = A S_i$

$N_o = A N_i + N_s$

$$F = \frac{S_i}{N_i} \times \frac{N_o}{S_o}$$

$$F = \left(\frac{S_i/N_i}{S_o/N_o}\right) = \frac{\text{Input SNR}}{\text{Output SNR}}$$

$$= \frac{S_i}{N_i} \times \frac{A N_i + N_s}{A S_i}$$

$$\boxed{F = 1 + \frac{N_s}{A N_i}}$$

If $N_s = 0$; CS is noiseless

$F_{min} = 1$

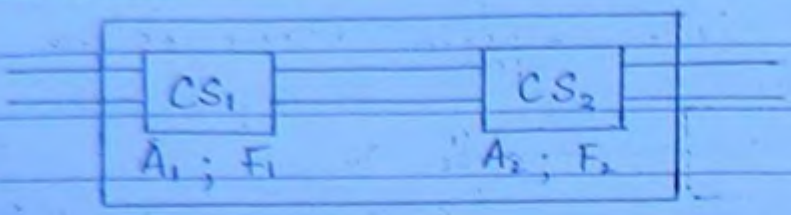
If $N_s \neq 0$; CS is noisy

$F > 1$

$$\boxed{F \geq 1}$$

$$F_{dB} = 10 \log_{10} F \text{ ... dB}$$

Cascaded CS



$$F = F_1 + \frac{F_2 - 1}{A_1}$$

for 2 cascaded communication system

under matched condition

$$G_n(f) = 2kTR$$

$$\Rightarrow G_n(f) = \frac{2kTR}{4R} = \frac{kT}{2} \text{ W/Hz}$$

$$N = G_n(f) \cdot 2 \cdot \Delta f$$

$$= \frac{kT}{2} \cdot 2 \cdot \Delta f$$

$$N = kT\Delta f \text{ W}$$

Other definitions of F.

$$F = 1 + \frac{N_o}{AN_i}$$

$N = 4kTR\Delta f$

$N \propto T$

$N \propto R$

$$F = 1 + \frac{T_{eq}}{T_o}$$

$$F = 1 + \frac{T_{eq}}{R_s}$$

T_{eq} represents equivalent noise temperature & is a measure of total noise power contributed by a specified communication system.

Equivalent noise resistance R_{eq} is a measure of total noise power contributed by a given communication system.

Find the following quantities at the input of RF amplifier using a device that has 220 Ω equivalent input noise resistance & a 300 Ω source resistance. The BW of the amplifier is 6 MHz & the room temperature is 17°C. Calculate.

i) Noise power
ii) Rms noise V/g
iii) Noise figure
iv) Equivalent Noise Temp.

$R_s = 300\ \Omega$

$R_{eq} = 220\ \Omega$

$\Delta f = 6\ MHz$

$T_0 = 17 + 273$

$= 290^\circ K.$

i) $N = 4KT(R_{eq} + R_s)\Delta f \quad \ldots W$

$= 48\ pW$

ii) $V_{rms} = \sqrt{N} = 6.93\ \mu V$

iii) $F = 1 + \frac{R_{eq}}{R_s} = 1.733 \Rightarrow F_{dB} = 10 \log_{10} 1.733 =$

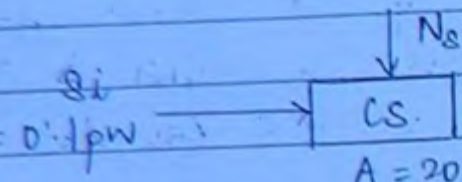
iv) $F = 1 + \frac{T_{eq}}{T_0} \Rightarrow (F-1)T_0 = T_{eq}$

$\rightarrow (0.733)\,290 = 212.57^\circ K. = T_{eq}$

2013

Q In a radio receiver the power gain of the 1st system is 20 & the noise generated by this stage is 0.04 pW. The signal power being 0.1 pW. The BW of the system is 500 KHz & room temp is 20°C. Find noise figure of this system. Repeat calculation for 2 identical cascaded stage.

(9)



$\Delta f = 500$ KHz.

$T = 20 + 273 = 293°K$.

$$F = 1 + \frac{N_e}{ANi} = 1 + \frac{N_s}{\frac{AkT}{2} \cdot 2\Delta f}$$

$$= 1 + \frac{0.04}{\frac{20 \times 293 \times 2 \times 500 \times 10^3 \times 1.23}{2}}$$

$10 \log_{10} 2 = 2$ dB

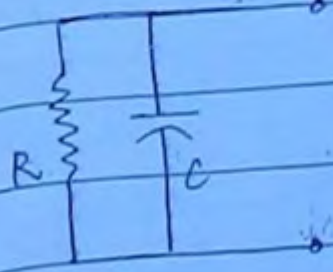
$$F = F_1 + \frac{F_2 - 1}{A_1} = 2 + \frac{2-1}{20} = 2 + 0.05 = 2.05$$

The noise figure of the system does not depend upon the signal power present at the input of the system.

By cascading power gain of the system increases to a very high value but there is only marginal increment of the contribution of noise.

A resistance of 1 kΩ is maintained at 300 K & is shunted by a capacitor of 1 μF. Find rms voltage across the capacitor over entire frequency band. What fraction of it is due to the frequencies between the half power frequencies.

(9)



$R = 1\,K$

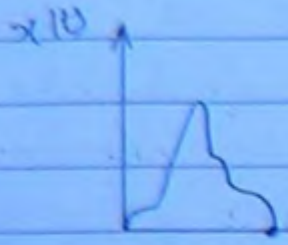
$C = 1\,\mu F$

$T = 300\ K$

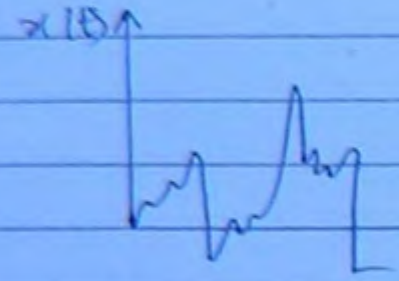
$$V_{rms} = \sqrt{\frac{KT}{C}} = 0.0645\,\mu V$$

$$V'_{rms} = \frac{V_{rms}}{\sqrt{2}} = \sqrt{\frac{KT}{2C}}$$

$$= 0.0456\,\mu V$$



... Energy signal
- Energy spectral density $G_E(\omega)$
- ... Energy contained per unit BW.
- ∴ J/Hz



... Power signal
- .. Power spectral density $G_P(\omega)$.
- .. Power contained per unit BW.
- ... W/Hz.

Representation of Random Signals –

1. Using probability density function ; pdf ; $p(x)$
2. Using auto-correlation function ; ACF ; $R(\tau)$; $\tau = t_2 - t_1$
3. Using spectral density ; PSD ; $G(\omega)$

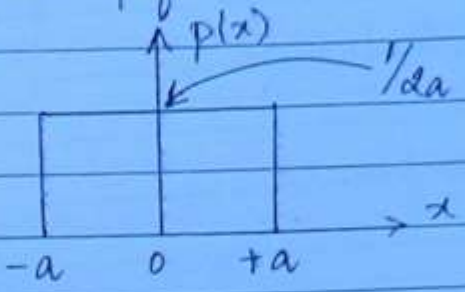
$$FT[R(\tau)] = G(\omega)$$
$$FT^{-1}[G(\omega)] = R(\tau)$$

$R(\tau) \rightleftharpoons G(\omega)$ Weiner Kintchine Theorem

Probability Density Functions – (pdf)

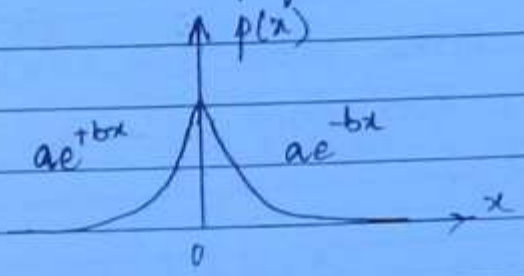
Various types :

1. Uniform pdf –



$$p(x) = \frac{1}{2a} \; ; \; -a < x < +a$$
$$= 0 \; ; \text{ otherwise}$$

2. Exponential pdf –



$$p(x) = ae^{+bx} \; ; \; x < 0$$
$$= ae^{-bx} \; ; \; x > 0$$

OR

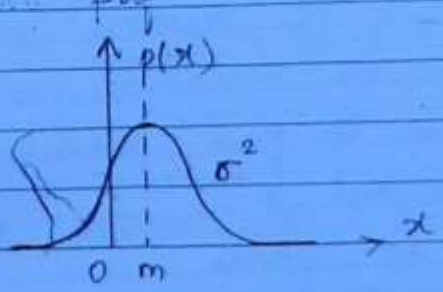
$$p(x) = ae^{-b|x|} \; ; \text{ for all } x$$

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\int_{-\infty}^{0} ae^{+bx}\,dx + \int_{0}^{\infty} ae^{-bx}\,dx = 1$$

find relation between 'a' & 'b'

3. Gaussian pdf –



$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-m)^2/2\sigma^2} \; ; \text{ for all } x$$

$$p(x) : G(m ; \sigma^2)$$

Significance of $p(x)$

1. mean ; 1st moment
   $m = \bar{x}$ $E(x)$

   $= \int_{-\infty}^{\infty} x\, p(x)\, dx$ .... dc voltage contained

2. $P_{dc} = m^2$ .... dc power contained.

3. 2nd moment :

   $\overline{x^{2}} = E(x^2)$

   $= \int_{-\infty}^{\infty} x^2\, p(x)\, dx$ .... total power (ac + dc) contained

4. Variance

   $\sigma^2 = \overline{x^2} - m^2$ ... ac power contained

5. $V_{rms} = \sqrt{\overline{x^2}}$ .... rms voltage contained

6. Standard deviation

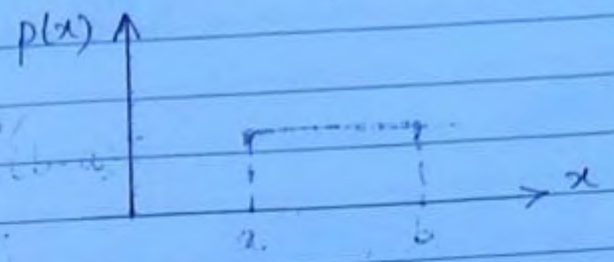
   s.d $\sqrt{\sigma^2}$ .... deviation of signal from its average value.

* Any random signal cannot be written in the mathematical form since its variation is purely random but we are able to find following features contained in the given signal

1. dc voltage
2. dc power
3. total power
4. ac power
5. rms value
6. deviation of the given random signal from its

A random signal has a uniform pdf between 'a' & 'b'. Find various features contained in the signal.

(95)



$$p(x) = \frac{1}{b-a} \qquad a < x < b$$

$$= 0 \qquad \text{otherwise}$$

1. dc voltage

$$m = \int_{-\infty}^{\infty} x \cdot p(x)\, dx = \int_{a}^{b} x \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{2}(b+a)$$

2. total power (ac + dc)

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2 \cdot p(x)\, dx$$

$$= \int_{a}^{b} x^2 \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{3}(a^2 + ab + b^2)$$

3. dc power $P_{dc} = m^2$

4. ac power $\sigma^2 = P_{ac} = P_{total} - P_{dc}$

$$= \overline{x^2} - m^2$$

5. $V_{rms} = \sqrt{\overline{x^2}}$

6. sd $\sqrt{\sigma^2}$

Auto correlation function

ACF ; $R(\tau)$

$\tau = t_2 - t_1$

$x(t+\tau)$ or $x(t)$ are even function

$$R(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt$$ ... for energy (pulse type) signals.

$$R(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} x(t)\, x(t-\tau)\, dt$$ ... for power (periodic type) signals

$\tau = 0$

$$R(0) = \int_{-\infty}^{\infty} x^2(t)\, dt \equiv E$$

$$R(0) = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} x^2(t)\, dt \equiv P$$

* Auto correlation function represents the sum of all the common properties between 1 signal & its shifted version

* ACF is a measure of the regularity with which a particular signal will exist. Higher is the occurance of regularity, higher is the value of ACF & vice versa.

* For $\tau = 0$ $R(0)$ represents the energy or power contained in the given signal depending upon type of signal.

Ex. $x(t) = A \sin \omega_0 t$ .... Periodic signal
.... Power signal.

To find 1. $R(\tau)$
2. P
3. features

Sol. $$R(\tau) = \frac{1}{T_0} \int_0^{T_0} x(t)\, x(t-\tau)\, dt$$

$$= \frac{1}{T_0} A^2 \int_0^{T_0} \sin \omega_0 t \sin \omega_0 (t-\tau)\, dt$$

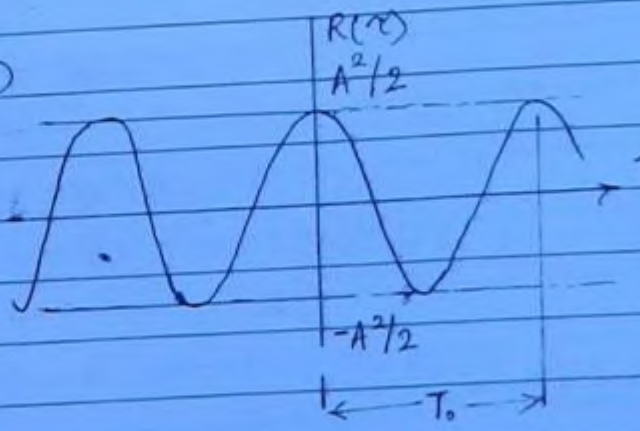
$$\frac{A^2}{2T_0} \int_0^{T_0} [\cos \omega_0 \tau - \cos \omega_0 (2t - \tau)]\, dt$$

$$= \frac{A^2}{2T_0} [T_0 \cos \omega_0 \tau - 0]$$

$$\boxed{R(\tau) = \frac{1}{2} A^2 \cos \omega_0 \tau} \quad \text{— (1)}$$

$$R(0) = P = \frac{1}{2} A^2 \quad \text{— (2)}$$

(3)



$R(\tau) = R(-\tau)$
Even function
$R(0) \geq |R(\tau)|$

$$\omega_0 = \frac{2\pi}{T_0}$$

Features :-

1. The ACF is always an even function of τ. (98)

2. For $\tau = 0$, $R(0)$ represents total power contained in the given signal.

3. The magnitude of $R(0)$ is always greater than or equal to the magnitude of $R(\tau)$ for some other value of τ.

4. For $\tau = 0$ $R(\tau)$ always has a maximum value & is repeated for every time interval T_0.

5. If given signal is periodic then its ACF is also periodic with same time period T_0 & same fundamental frequency ω_0.

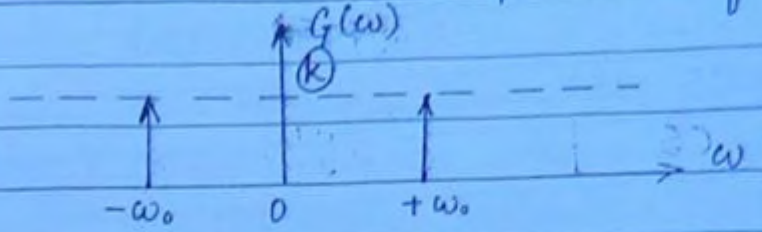
Ex. $$R(\tau) = \frac{1}{2} A^2 \cos \omega_0 \tau$$

Find
1. $G(\omega)$; PSD
2. P

Sol. (1) $$G(\omega) = FT[R(\tau)]$$
$$= \frac{1}{2} A^2 \, FT[\cos \omega_0 \tau]$$
$$= \frac{1}{2} A^2 \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

$\rightarrow$ PSD

→ always an even function of ω.



$$k = \frac{1}{2} A^2 \pi$$

99

(2) $$P = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \cdot \frac{1}{2} A^2 \pi \int_{-\infty}^{\infty} [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]\, d\omega$$

$$= \frac{A^2}{4} [1 + 1]$$

$$P = \frac{1}{2} A^2$$

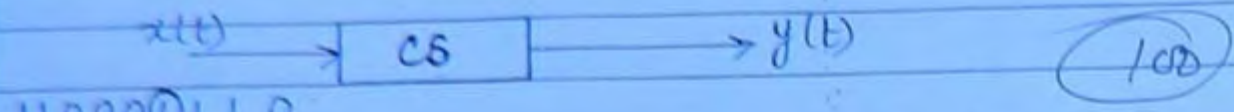
Summary -

$x(t)$ —
- pdf = $p(x)$
- $R(\tau)$
- PSD ; $G(\omega)$

To find : P

1. $\overline{x^2(t)} = P$
2. $P = \overline{X^2} = \int_{-\infty}^{\infty} x^2 p(x)\, dx$
3. $P = R(\tau)\Big|_{\tau=0} = R(0)$
4. $P = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)\, d\omega$

INFORMATION THEORY



100

11000110

-- bit stream

x_i

Information associated with transmission of x_i digit

A

$$I(x_i) = \log \frac{1}{P(x_i)} = -\log P(x_i)$$

- units of information -

$$I(x_i) = -\log_2 P(x_i) \text{ ... bits}$$
$$= -\log_e P(x_i) \text{ ... nits}$$
$$= -\log_{10} P(x_i) \text{ ... Hartley}$$

1. The information associated with the transmission of any digit depends upon the probability with which it occurs. Higher is the probability of occurance lower is the ~~base~~ information associated with it.

2. Units of information depends upon the base of the logs. unless specified we always take log to the base 2

Channel Capacity [C] -

The rate at which the data can be transmitted over any communication channel must always be maximum to effectively utilise that channel.

The channel capacity represents the maximum rate at which the data can be transmitted over the communication channel. This transmission of data depends upon -

i) losses in the channel OR SNR available on the channel
ii) availability of the B.W. of the system. The channel capacity is given by the Shannon - Hartley theorem.

$$\boxed{C = B \log_2 \left(1 + \frac{S}{N}\right)} \quad \text{... bps}$$

B = channel BW

S/N = Signal to noise ratio available on the channel.

Ex. Find the information associated with a digit which is occuring with a probability of 1/6.

Sol $P(x_i) = \frac{1}{6}$

$$I(x_i) = -\log_2 P(x_i) = -\log_2 \frac{1}{6}$$

$$= \log_2 6 \quad \text{bpc}$$

$$= \frac{\log_{10} 6}{\log_{10} 2}$$

$$= 2.58 \text{ bps}$$

Ex. Voice channel of a telephone n/w has a BW of 3.4 KHz. Find -
i) channel capacity for a SNR of 30 dB
ii) min SNR required to support the information at a rate of 4.8 Kbps.

Sol (i) $C = B \log_2\left(1 + \frac{S}{N}\right)$

~~$= 3.4 \times 10^3 \log_2(1 + 14.77)$~~

~~$= 13.53$~~

$\frac{S}{N} = 30$ dB

$\Rightarrow 10 \log_{10}\left(\frac{S}{N}\right) = 30$

$\frac{S}{N} = 10^3$

$C = 3.4 \log_2(1001)$

$= \frac{3.4 \log_{10}(1001)}{0.3010}$

$= 33.89$ kbps.

(ii) $C = 4.8$ kbps.
$B = 3.4$ kHz

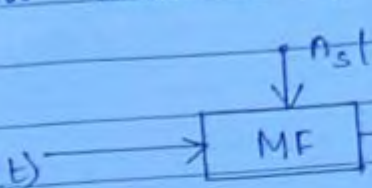
$4.8 = 3.4 \log_2\left(1 + \frac{S}{N}\right)$

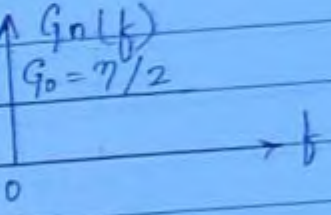
$1.412 = \frac{1}{0.3010} \log_{10}\left(1 + \frac{S}{N}\right)$

$0.4249 = \log_{10}(1 + S/N)$

$\frac{S}{N} \simeq 1.66$

Matched Filter (MF) -





$$G_n(f) = \frac{\eta}{2} \quad ; \quad -\infty < f < +\infty$$

Transfer function of MF $\equiv H(\omega)$
Impulse response $= h(t)$

* A matched communication system OR a matched filter maximises SNR at its output in the presence of white noise

* This system has more relevance for digital communication systems

Features of MF -

1. Max SNR at the output

$$\rho_{max} = \left(\frac{S_o}{N_o}\right)_{max} = \frac{2E}{\eta} = \frac{E}{\eta/2} = \frac{\text{Energy in input signal}}{\text{PSD of White noise}}$$

2. Impulse response

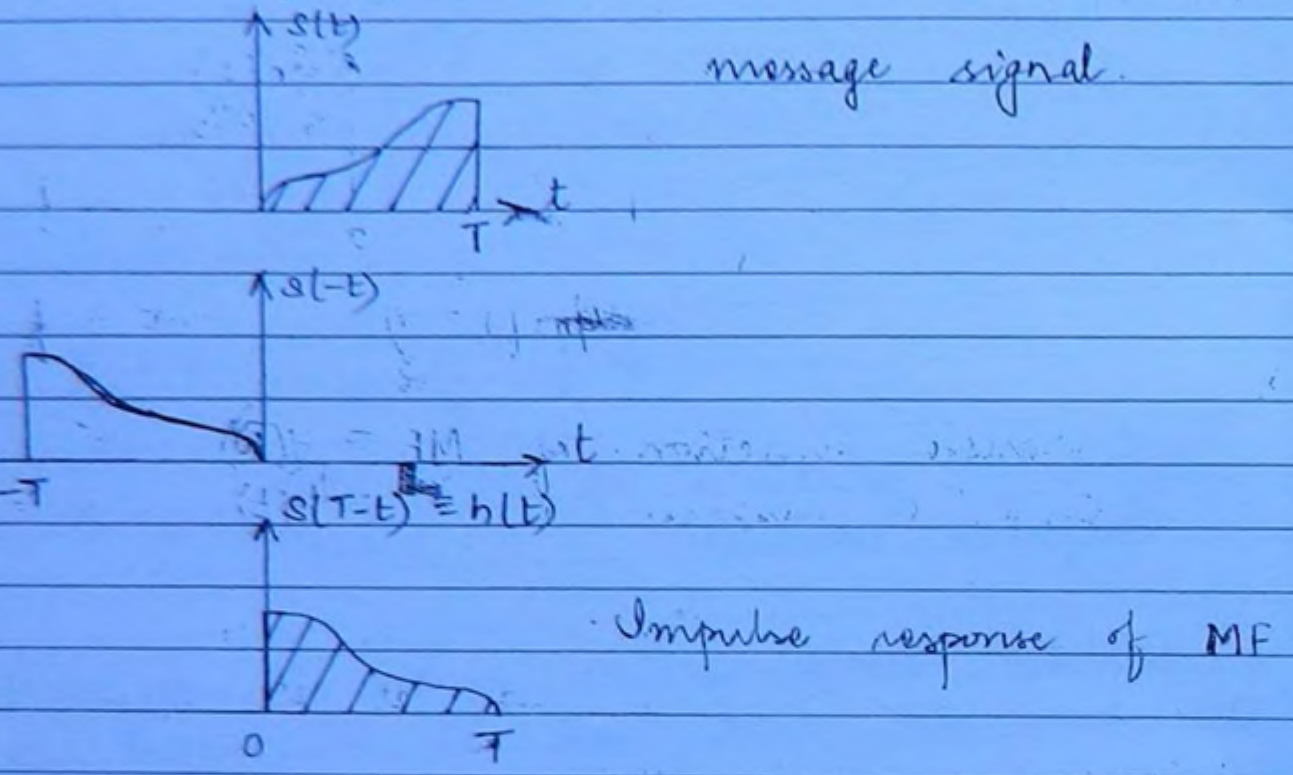
$$h(t) = s(T - t)$$

T - time instant at which SNR is maximised

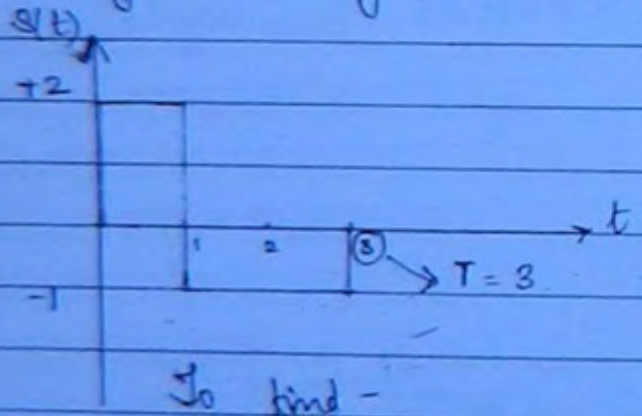
3. Transfer function

$$H(\omega) = FT[h(t)]$$
$$= S(-\omega) e^{-j\omega T}$$

Significance of $h(t) = s(T-t)$



Ex. The signal is given as -



To find -

1) $f_{max} = \left(\frac{S_0}{N_0}\right)_{max} = \frac{2E}{\eta}$

2) $h(t) = s(T-t)$

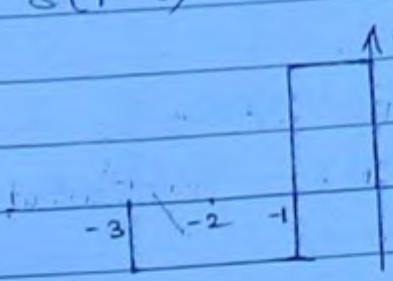
3) $H(\omega) \equiv FT[h(t)] = S(-\omega) e^{-j\omega T}$

(1) $E = \int_{-\infty}^{\infty} s^2(t)\,dt = \int_0^1 (2)^2\,dt + \int_1^3 (-1)^2\,dt = 4 + 2 = 6\ J.$

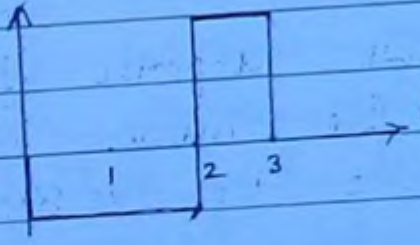
(105)

(ii) $\left(\frac{S_o}{N_o}\right)_{max} = \frac{2E}{\eta} = \frac{12}{\eta}$

(2) $h(t) = s(T-t)$



$s(T-t) = h(t)$



Impulse response of MF.

(3) $h(t) = -1 \quad 0 < t < 2$

$= +2 \quad 2 < t < 3$

$H(\omega) = FT[h(t)] = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\,dt$

$= \int_0^2 (-1) e^{-j\omega t}\,dt + \int_2^3 (2) e^{-j\omega t}\,dt$

$= \frac{e^{-j\omega t}}{j\omega}\Big|_0^2 + \frac{(-2)\, e^{-j\omega t}}{j\omega}\Big|_2^3$

$= e^{-2j\omega} - 1 - 2(e^{-3j\omega} - e^{-2j\omega}) \div (j\omega)$

$= (e^{-2j\omega} - 1 - 2e^{-3j\omega} + 2e^{-2j\omega}) / (j\omega)$

$= \frac{3e^{-2j\omega} - 2e^{-3j\omega} - 1}{j\omega}$

Summary -

1. $\left(\frac{S}{N_0}\right)_{max} = \frac{2E}{\eta}$
2. $h(t) = s(T-t)$
3. $H(\omega) = S(-\omega) e^{-j\omega T}$

1. Maximum SNR at the output of a MF depends only upon energy contained in the input signal.

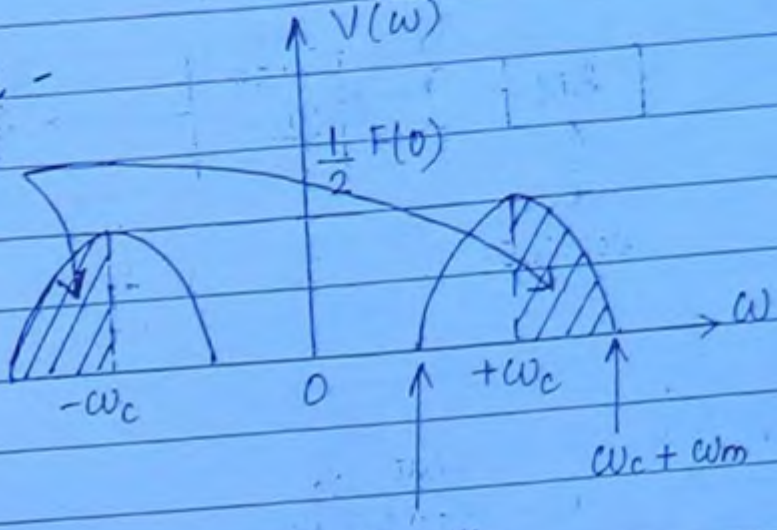
2. The impulse response function of a MF represents the mirror image of input message signal about the vertical axis & shifted to the right by T seconds.

3. The transfer function of the MF depends only upon shape of input message signal.

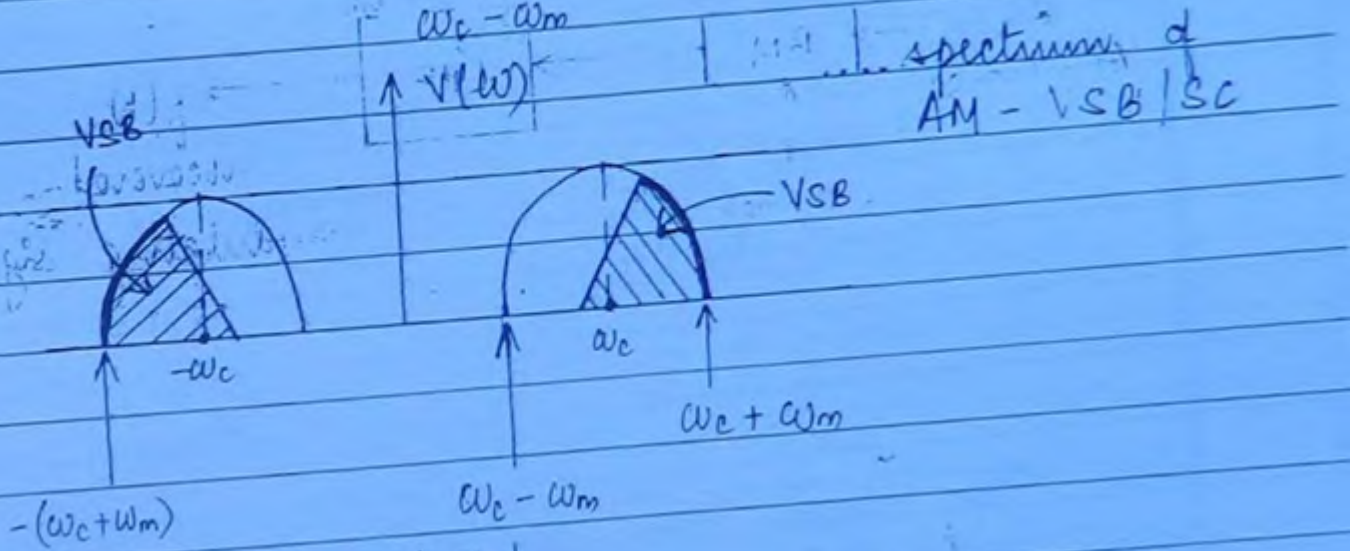
VSB Transmission

... Vestigial Side Band Transmission.

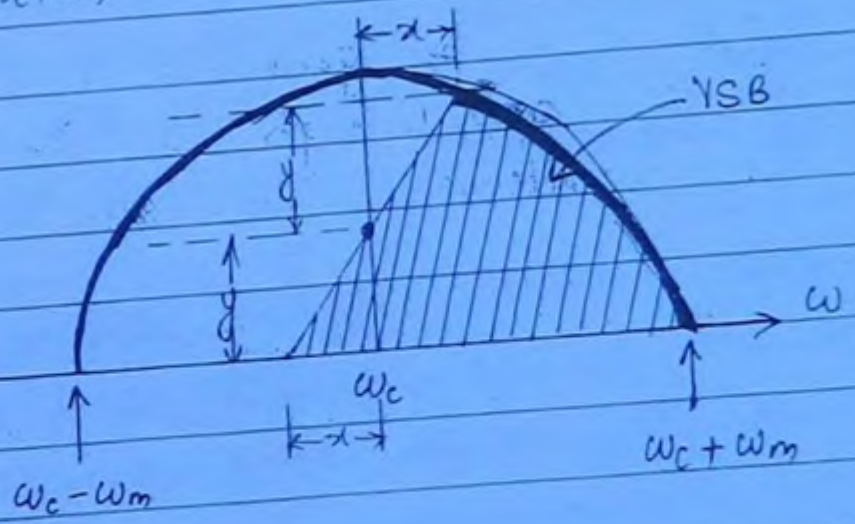
Principle -



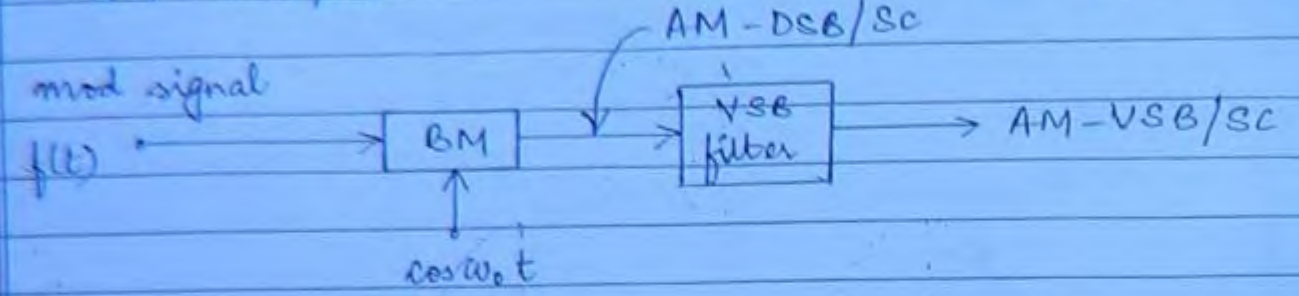
.... AM - DSB/SC spectrum



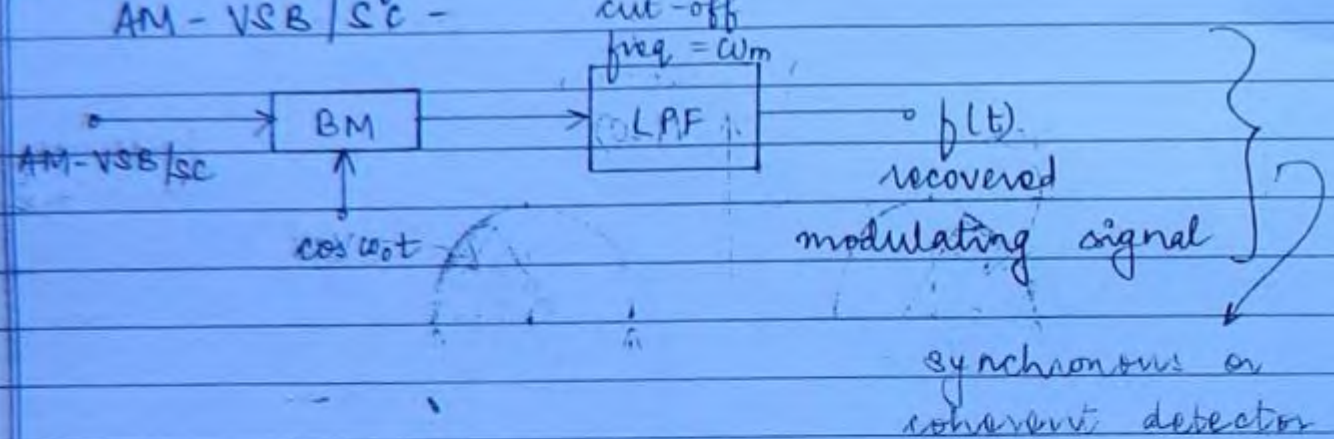
... spectrum of AM - VSB/SC



Generation of AM - VSB/SC -



Demodulation of AM - VSB/SC -



The VSB signal can be generated by passing the DSB signal through a VSB filter whose cut-off characteristics may not be sharp

The VSB signal practically required requires almost same BW as that of SSB signal but its generation is very simple using a practical filter.

The VSB filter must have such characteristics so that those freq components which have been attenuated in 1 SB the same freq components are allowed to pass in the other SB with the same magnitude so that there is no loss of information

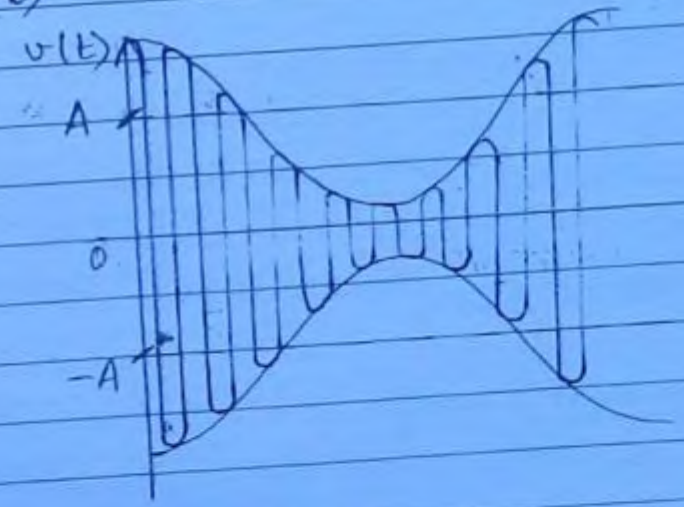
AM-VSB/FC system is used for television broadcast for transmission of video signal.

for SC signal original modulating signal is recovered by using synchronous or coherent detection whereas the FC signal can be demodulated using envelope detection.

Choice of RC time constant for Envelope Detector (AM-DSB/FC)

.... Proof of

$$\frac{1}{RC} \geq \frac{\omega_m m_a}{\sqrt{1-m_a^2}}$$



$$v(t) = \underbrace{A(1+m_a\cos\omega_m t)}_{\text{Envelope of composite signal } e(t)}\cos\omega_c t$$

$$e(t) = \underbrace{A(1+m_a\cos\omega_m t)}_{\text{slope of envelope}} = E_0$$

$$\frac{-de(t)}{dt} = -Am_a\omega_m \sin\omega_m t \quad \text{---(1)}$$

Capacitor voltage

$$e_c(t) = E_0 e^{-t/RC}$$

Slope

$$\frac{-de_c(t)}{dt} = \frac{E_0}{RC} e^{-t/RC} \Big|_{t=0}$$

$$= \frac{E_0}{RC}$$

$$\frac{-de_c(t)}{dt} = \frac{A}{RC}(1 + m_a \cos \omega_m t) \quad \text{---- (2)}$$

So avoid diagonal clipping -

$$\left|\frac{de_c}{dt}\right| \geq \left|\frac{de}{dt}\right|$$

$$\frac{A}{RC}(1 + m_a \cos \omega_m t) \geq A m_a \omega_m \sin \omega_m t$$

$$\frac{1}{RC} \geq \underbrace{\frac{\omega_m m_a \sin \omega_m t}{1 + m_a \cos \omega_m t}}_{\text{must be max}}$$

$$\frac{d}{dt}\left[\frac{\omega_m m_a \sin \omega_m t}{1 + m_a \cos \omega_m t}\right] = 0$$

$$\cos \omega_m t = -m_a$$

$$\Rightarrow \sin \omega_m t = \sqrt{1 - m_a^2}$$

$$\frac{1}{RC} \geq \frac{\omega_m m_a \sqrt{1 - m_a^2}}{1 + m_a(-m_a)}$$

$$\boxed{\frac{1}{RC} \geq \frac{\omega_m m_a}{\sqrt{1 - m_a^2}}}$$ ... RC time constant for Envelope detector.

Central Limit Theorem -

(111)

If $X = X_1 + X_2 + \dots$ } $X_1, X_2 \dots$ random processes

then $m = m_1 + m_2 + \dots$

$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \dots$ having any type of pdf

X... always is Gaussian with mean m Variance σ^2

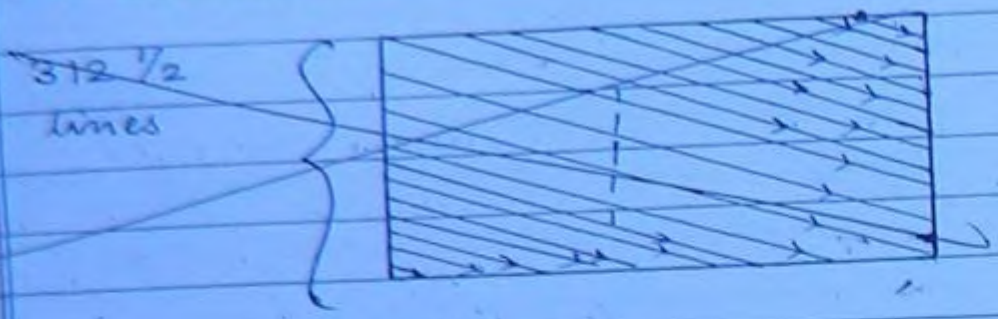
$\Rightarrow X : \underbrace{G(m ; \sigma^2)}$

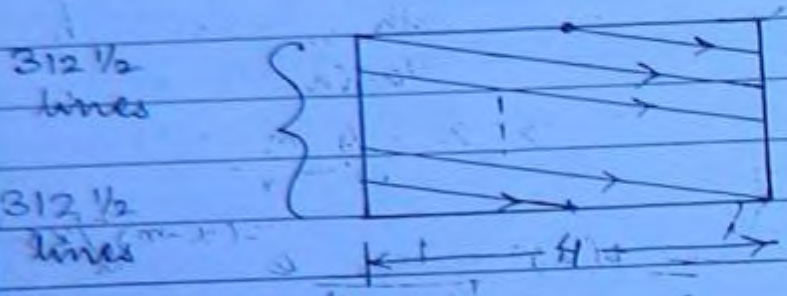
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-m)^2/2\sigma^2}$$

* When small n random processes are added to obtain a new random process then PDF of the resulting random process will always be Gaussian with mean m & variance σ^2. The result is applicable inspective of type of pdf of the individual process.

* The result is again applicable inspective of whether n random processes are statistically independent or not.

TV standards :-





... inter-laced scanning

⇒ 625 lines / frames
25 frames / sec

aspect ratio
4 : 3

No. of lines per frame is equal to 625.

No. of frames per second is equal to 25.

Field frequency = 50

Line frequency = 625 lines × 25 lines / frame
= 15,625 lines / sec

Channel Bandwidth = 7 MHz

Video Bandwidth = 5 MHz

Colour subcarrier = 4.43 MHz

Video system is AM

Audio system is FM

Maximum audio deviation $\delta = \pm 50$ KHz

Interlace ratio = 2:1 & represents the field frequency ÷ frame frequency.

Aspect ratio = 4:3 & represents the ratio of horizontal distance to the vertical distance.

Type of modulation →

Picture signal : AM - VSB | FC
with most of LSB suppressed.

Audio signal : WBFM.

Modulation polarity -
In modulation polarity negative video modulation where black corresponds to higher modulation percentage than white.

* Synchronisation -

Synchronisation pulse - are transmitted along with picture information. These two set of signals are then TDMed & picture carrier is amplitude modulated by this total information. This ensures that the receiver picture tube is synchronised with the transmitter camera tube